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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two questions

1 a) If the random variable X takes the values 1,2,3 and 4 such that 2P(X=1)=3P(X=2)=P(X=3)=5P(X=4), find the probability distribution and cumulative distribution function of X 7

b) A complex electronic system is built with a certain number of backup components in itssubsystems. One subsystem has four identical components, each with a probability of 0.2 offailing in less than 1000 hours. The subsystem will operate if any two of the four componentsare operating. Assume that the components operate independently. Find the probability that 8

i)exactly two of the four components last longer than 1000 hours.

ii)the subsystem operates longer than 1000 hours.

2 a) A gardener sows 4 seeds in each of 100 plant pots. The number of pots in which 0,1,2,3 and 4 of seeds germinated is given in the following table. Fit a binomial distribution to the data 7

No. of seeds germinated	0	1	2	3	4
No. of pots	13	35	34	15	3

b) Find the mean and variance for the pdf 8

$$f(x) = \begin{cases} kx^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

3 a) If a random variable X has the exponential distribution with mean ½, calculate the probabilities that i) X will lie between 1 and 3 ii) X is greater than 0.5 iii) X is atmost 4 7

b) In a normal distribution, 10% of the items are below 55 and 20% are above 59. Find the mean and standard deviation of the distribution. What percentage of the items are above 60? 8

PART B

Answer any two questions

4 a) Obtain the distribution function of a continuous two dimensional random variable (X,Y) with the joint pdf given by 7

$$f(x,y) = \begin{cases} e^{-x-y}, & 0 < x < \infty, \ 0 < y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

b) If the joint pdf of (X,Y) is given by 8

$$f(x,y) = \begin{cases} \frac{1}{2}, & x > 0, y > 0, x + y < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find $P\{X \leq 1, Y \leq 1\}$, $P\{X + Y < 1\}$ and $P\{X > 2Y\}$

5 a) The joint pdf of (X,Y) is 7

$$f(x,y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

i) Check whether X and Y are independent

ii) Find $P(X + Y < 1)$

b) Prove that the power spectral density and autocorrelation function of a real WSS process form a Fourier cosine transform pair 8

6 a) If $X(t) = P + Qt$ is the random process where P and Q are independent random variables with $E(P)= p$, $E(Q)= q$, $Var(P)= \sigma_1^2$, $Var(Q) = \sigma_2^2$ then find the mean, autocorrelation and autocovariance of the process 7

b) Consider the random process $X(t)= A\cos(\omega t) + B\sin(\omega t)$ where A and B are independent random variables with mean 0 and equal variance. Show that X(t) is a WSS. 8

PART C

Answer any two questions

7 a) Cell phone calls processed by a certain wireless base station arrive according to a Poisson process with an average of 12 per minute. i) What is the probability that more than two calls arrive in an interval of length 20 seconds ii) What is the probability that more than 2 calls arrive in each of two consecutive intervals of length 30 seconds 7

b) Show that the time between any two consecutive occurrences of a Poisson process is a random variable following an exponential distribution 7

c) Find the mean, variance, autocorrelation and autocovariance of a Poisson process 6

8 a) Find the positive solution of the equation 2sinx = x using Newton-Raphson method 6

b) Using Newton's forward difference interpolation formula evaluate f(2.05) from the following table 6

X	2.0	2.1	2.2	2.3	2.4
f(x)	1.414214	1.449138	1.483240	1.516575	1.549193

c) Find an approximate value of $\log_e 5$ by evaluating $\int_0^5 \frac{dx}{4x+5}$ using Simpson's 1/3rd rule with h=0.5 8

9 a) The transition probability matrix of a Markov chain $\{X_n\}$, n =1,2,3,..... having three states 1,2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 04 \\ 0.3 & 0.4 & 0.3 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$ 10

And the initial distribution is

$P(0)=[\,0.7 \;\; 0.2 \;\; 0.1\,]$. Find $P(X_2=3)$

b) Evaluate $\int_0^1 \frac{dx}{1+x}$ using trapezoidal rule 5

c) Using Runge-Kutta method of order four, find y(0.2) given that $\frac{dy}{dx} = y - x$, y(0)=2 by taking h=0.1 5