B

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

Branch: Civil Engineering

Stream: Structural Engineering

01CE6103 Theory of Elasticity

Answer *any two full* questions from *each* part

Limit answers to the required points.

Max. Marks: 60   Duration: 3 hours

PART A

1.   a.   Explain Lame's stress ellipsoid and stress director surface   4

     b.   At a certain point in a drive shaft coupling, the stress components relative to   5
axes(x,y,z) are
$\sigma_{xx} = 80MPa, \sigma_{yy} = 60MPa, \sigma_{zz} = 20MPa, \tau_{xy} = 20MPa, \tau_{xz} = 40MPa, \tau_{yz} = 10MPa$

        (a) Determine the stress vector on a plane normal to the vector R= i+2j+k
        (b) Determine the principal stresses and the maximum shear stress

     Determine the octahedral shear stress and compare it to the maximum shear
stress

2.   a.   Derive Lame -Navier's equations in 3D.   3

     b.   6

      The strain tensor at a point in a body is given by $[\varepsilon] = \begin{bmatrix} 12 & 3 & 4 \\ 3 & 8 & -4 \\ 4 & -4 & 18 \end{bmatrix} \times 10^{-3}$

        (i)   Determine the normal strain along a direction whose direction
cosines with respect to the coordinate directions are given by
$\ell = m = n = \dfrac{1}{\sqrt{3}}$

        (ii)   Determine the principal strains and principal axes

3.   Write notes on   9

        (i)   The significance of compatibility equations
        (ii)   Boundary value problems of elasticity
        (iii)   Hydrostatic and deviatoric strains

PART B

4.   a.   State and prove uniqueness theorem in theory of elasticity   4

b. Discuss generalized Hooke's law and hence on the reduction in number of elastic constants for different material properties   5

5. Using stress function approach, derive the expression for the maximum deflection of a simply supported beam of length 'ℓ' and depth '2c' if the beam is subjected to a uniformly distributed load of 'q'   9

6. a. Derive Navier's equations for a three - dimensional stress state   4

   b. Show that the function $\phi = A\left(xy^3 - \frac{3}{4}xyh^2 \right)$ is an Airy's stress function and hence show that it represents the stress distribution in a cantilever beam loaded at the free end with a load P. Find the value of A if the width of beam is 'b' and depth is 'h'. Assume $\tau_{xy} = 0\, at\ y = \pm \frac{h}{2}$ 5

PART C

7. Show that the stress concentration around a hole in a plate of infinite dimension under uni-axial tension is 3. Plot the variation of stresses around the hole.   12

8. a. Derive from first principles, the equations of compatibility in polar co-ordinates for a two- dimensional problem   6

   b. Derive the expressions for the stresses in a solid cylinder rotating with an angular speed of 'ω' and having a mass density of 'ρ'   6

9. a. Derive the expression for the maximum shear stress in an elliptical bar due to torsion   7

   b. A hollow circular member subjected to torsion has an outside diameter of 22mm and an inside diameter of 18mm, with mean diameter D=20mm and t/D=0.1. Calculate the torque and angle of twist per unit length if the shearing stress at mean diameter is 70 MPa. Calculate these values if a cut is made through the wall thickness along the entire length. G=77.5 GPa   5