

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

Electrical & Electronics Engineering
01MA6021: ADVANCED MATHEMATICS AND OPTIMIZATION TECHNIQUES
(Common to all streams)

Answer *any two full* questions from *each* part

Limit answers to the required points.

Max. Marks: 60

Duration: 3 hours

PART A(Module I and II)

1. a. Find the dimension and a basis for the subspace of \mathbb{R}^3 spanned by the vectors $(-1, 2, 1), (2, -1, 0)$, and $(1, 4, 3)$ (4 marks)
1. b. Find a basis for $\text{Col } A$; if matrix $A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ -3 & 1 & -4 & -5 \\ -1 & -4 & 5 & -6 \\ 2 & 1 & 1 & 5 \end{bmatrix}$ (5 marks)
2. a. Let W be a subspace of \mathbb{R}^n ; prove that its orthogonal complement W^\perp is a subspace of \mathbb{R}^n . (4 marks)
2. b. Determine the dimensions of the kernel and range of the transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ by $T(x, y, z) = (x + y, z)$. (5 marks)
3. a. Find a least square solution of $AX = b$; where

$$A = \begin{bmatrix} 1 & 5 \\ 3 & 1 \\ -2 & 4 \end{bmatrix}; \quad b = \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix}$$
 (4 marks)
3. b. Find the singular value decomposition of $A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$. (5 marks)

PART B(Module III & IV)

4. Using Two Phase Simplex method, solve

Maximize $Z = 3x_1 - x_2$ (9 marks)

Subject to $2x_1 + x_2 \geq 2$; $x_1 + 3x_2 \leq 2$; $x_2 \leq 4$;

$$x_1, x_2 \geq 0$$

5. a. Write the dual of the following LPP(4 marks)

$$\text{Maximize } f = 4x_1 + 2x_2$$

Subject to $x_1 - 2x_2 \leq 2$; $x_1 + 2x_2 = 8$; $x_1 - x_2 \leq 11$;

$x_1 \geq 0$; x_2 unrestricted.

b. Solve by Graphical Method(5 marks)

$$\text{Maximize } Z = 7x_1 + 3x_2$$

Subject to $x_1 + 2x_2 \geq 3$; $x_1 + x_2 \leq 4$; $x_1 \leq \frac{5}{2}$; $x_2 \geq \frac{3}{2}$;

$x_1, x_2 \geq 0$; x_1, x_2 integers.

6. Solve the Integer Programming Problem by Cutting plane method (9 marks)

$$\text{Maximize } Z = 2x + 3y$$

Subject to $2x + 2y \leq 7$; $x \leq 2$; $y \leq 2$;

$x, y \geq 0$; x and y integers.

PART C (MODULE V & VI)

7. a. Minimize $f = x_1^2 + 2x_2^2$ from the starting point (1,1), using Steepest Descent Method. (Two Iterations) (6 marks)

b. Write the Kuhn-Tucker conditions for

$$\text{Minimize } f = x_1^2 + 2x_2^2 + 3x_3^2;$$

subject to $x_1 - x_2 - 2x_3 \leq 12$; $x_1 + 2x_2 - 3x_3 \leq 8$. (6 marks)

8. a. Write down the iterative procedure of Fletcher Reeves Method? (6 marks)

b. Use Hooke Jeeves method to solve

$$\text{Minimize } f(X) = 3x_1^2 + x_2^2 - 12x_1 - 8x_2; \text{ with } X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \Delta x_1 = \Delta x_2 = 0.5.$$

9. a. Write the necessary conditions for the optimal solution of the QPP(6 marks)

$$\text{Min } f(x) = 3x_1^2 + x_2^2 + 2x_1x_2 + x_1 + 6x_2 + 2$$

subject to $2x_1 + 3x_2 \geq 4$; $x_1, x_2 \geq 0$.

b. Determine u_1, u_2, u_3 so as to maximize $u_1u_2u_3$ subject to $u_1 + u_2 + u_3 = 10$ and $u_1, u_2, u_3 \geq 0$. (6 marks)