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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION DECEMBER 2015

Electrical and Electronics Engineering

(COMMON to all streams)

01MA6021: Advanced Mathematics & Optimization Techniques

Time: 3 hours Max: Marks : 60

Answer any two full questions from each part.

PART–A (Module: I and II)

1. a. Determine whether $S = \{(x_1, x_2, x_3) | x_i \geq 0, x_i \in R^3\}$ is a subspace of R^3. Justify your answer. (4)

b. Find a basis for the null space and column space of $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$ (5)

2. a. Let T be defined by $T(x, y) = (3x + y, 5x + 7y, x + 3y)$. Show that T is a one to one linear transformation. Does T map R^2 onto R^3 (4)

b. Let U be the sub space of R^3 spanned by the vectors $u_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

Find an orthonormal basis for U by Gram-Schmidt orthogonalization process. (5)

3. a. Find a singular value decomposition of $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$ (5)

b. Find a least squares solution of the inconsistent system Ax = b where $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and

$b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$ (4)

PART-B (Module III and IV)

4. a. Solve the following LPP by simplex method. Minimize $f = -x_1 - 2x_2 - x_3$, subject to the constraints $2x_1 + x_2 - x_3 \leq 3$, $2x_1 - x_2 + 5x_3 \leq 6$, $4x_1 + x_2 + x_3 \leq 6$, $x_1, x_2, x_3 \geq 0$ (6)

b. Construct the dual of the LPP
Maximize $f = 50x_1 + 100x_2$ subject to the constraint $2x_1 + x_2 \leq 1250$, $2x_1 + 5x_2 \leq 1000$, $2x_1 + 3x_2 \leq 900$, $x_2 \leq 150$, $x_1, x_2 \geq 0$ (3)

5. a. How can you solve an integer non linear programming problem? (3)

b. Minimize $f(X) = x_1^2 - x_1x_2 + 3x_2^2$ starting at $X_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ by the method of steepest descent. (carry out only two iterations) (6)

6. a. What are the roles of exploratory and pattern moves in the Hook and Jeeves method? (3)

b. Minimize $f(X) = x_1^2 - x_1x_2 + x_1 + 3x_2 - 1$ starting at $X_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ by conjugate gradient method. (6)

PART-C (Module V and VI)

7. a. Consider the problem Minimize $f(\mathbf{X}) = (x_1 - 1)^2 + (x_2 - 5)^2$ subject to

$g_1 = -x_1^2 + x_2 - 4 \le 0$, $g_2 = -(x_1 - 2)^2 + x_2 - 3 \le 0$

Formulate the direction finding problem at $X = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$ as a linear programming problem in Zoutendijk's method. (6)

b. $\text{Min} f(\mathbf{X}) = x_1^2 + x_2^2 - 6x_1 - 8x_2 + 10$ subject to $4x_1^2 + x_2^2 \le 16$ $3x_1 + 5x_2 \le 0$ $x_1, x_2 \ge 0$ with starting point $X = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ using cutting plane method. Complete one step of the process. (6)

8. a. Apply Kuhn -Tucker condition to solve the following problem

$\text{Min } f(\mathbf{X}) = -2x_1 - x_2$ subject to $x_1 - x_2 \le 0$, $x_1^2 + x_2^2 \le 4$ $x_1, x_2 \ge 0$ (6)

b. Minimize $\text{Min } f(\mathbf{X}) = \frac{1}{3}(x_1 + 3)^2 + x_2^3$ subject to

$g_1(\mathbf{X}) = x_1 - 2 \ge 0$, $g_2(\mathbf{X}) = x_2 \ge 0$, by exterior penalty function method. (6)

9. a. Determine whether the following optimization problem is convex, concave or neither type $\text{Min } f(\mathbf{X}) = -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$, subject to $2x_1 + x_2 \le 6$, $x_1 - 4x_2 \le 0$, $x_1, x_2 \ge 0$ (6)

b. Solve the following Linear programming problem as a dynamic programming problem Maximize $z = 3x_1 + 4x_2$ subject to the constraint $2x_1 + x_2 \le 40$, $2x_1 + 5x_2 \le 180$, $x_1, x_2 \ge 0$ (6)
