

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
FIRST SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

**Course Code: MA101**

**Course Name: CALCULUS**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions, each carries 5 marks.*

- |   |  | Marks |
|---|--|-------|
| 1 | a) Determine whether the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$ converges. If so, find the sum   | (2)   |
|   | b) Examine the convergence of $\sum (\frac{k}{k+1})^{k^2}$   | (3)   |
| 2 | a) Find the slope of the surface $z = x e^{-y} + 5y$ in the y direction at the point (4, 0)  | (2)   |
|   | b) Show the function $f(x, y) = e^x \sin y + e^y \cos x$ satisfies the Laplace's equation $f_{xx} + f_{yy} = 0$  | (3)   |
| 3 | a) Find the directional derivative of $f(x, y, z) = x^3 z - yx^2 + z^2$ at P (2, -1, 1) in the direction of $3\vec{i} - \vec{j} + 2\vec{k}$  | (2)   |
|   | b) Find the unit tangent vector and unit normal vector to the curve $\mathbf{r}(t) = 4\cos t \mathbf{i} + 4 \sin t \mathbf{j} + t \mathbf{k}$ at $t = \frac{\pi}{2}$                         | (3)   |
| 4 | a) Using double integration, evaluate the area enclosed by the lines $x = 0, y = 0, \frac{x}{a} + \frac{y}{b} = 1$   | (2)   |
|   | b) Evaluate $\int_{-1}^2 \int_0^2 \int_0^1 (x^2 + y^2 + z^2) dx dy dz$   | (3)   |
| 5 | a) If $\mathbf{F}(x, y, z) = x^2 \mathbf{i} - 3\mathbf{j} + yz^2 \mathbf{k}$ find $\text{div } \mathbf{F}$   | (2)   |
|   | b) Find the work done by the force field $\mathbf{F} = xy \mathbf{i} + yz \mathbf{j} + zx \mathbf{k}$ on a particle that moves along the curve C: $x = t, y = t^2, z = t^3, 0 \leq t \leq 1$ | (3)   |
| 6 | a) Use Green's theorem to evaluate $\int_c (x dy - y dx)$ , where c is the circle $x^2 + y^2 = a^2$  | (2)   |
|   | b) If S is any closed surface enclosing a volume V and $\mathbf{F} = x\mathbf{i} + 2y\mathbf{j} + 3z\mathbf{k}$ show that $\iint_S \mathbf{F} \cdot \mathbf{n} ds = 6V$                      | (3)   |

**PART B**

**Module I**

*Answer any two questions, each carries 5 marks.*

- |   |   |     |
|---|---|-----|
| 7 | Determine whether the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+7}{k(k+4)}$ is absolutely convergent | (5) |
| 8 | Find the Taylor series expansion of $f(x) = \frac{1}{x+2}$ about $x = 1$  | (5) |
| 9 | Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(x+1)^k}{k}$      | (5) |

**Module II**

*Answer any two questions, each carries 5 marks.*

- |    |  |     |
|----|--|-----|
| 10 | Find the local linear approximation L to the function $f(x, y, z) = xyz$ at the point P (1, 2, 3). Also compare the error in approximating f by L at the point Q (1.001, 2.002, 3.003) with the distance PQ. | (5) |
| 11 | Locate all relative extrema and saddle points of $f(x, y) = 2xy - x^3 - y^2$   | (5) |
| 12 | If $u = f(\frac{x}{y}, \frac{y}{z}, \frac{z}{x})$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$                                       | (5) |

## Module III

*Answer any two questions, each carries 5 marks.*

- 13 Write the parametric equations of the tangent line to the graph of  $\mathbf{r}(t) = \ln t \mathbf{i} + e^{-t} \mathbf{j} + t^4 \mathbf{k}$  at  $t = 2$  (5)
- 14 A particle moves along the curve  $\mathbf{r} = (t^3 - 4t) \mathbf{i} + (t^2 + 4t) \mathbf{j} + (8t^2 - 3t^3) \mathbf{k}$  where  $t$  denotes time. Find (5)
- (i) the scalar tangential and normal components of acceleration at time  $t = 2$
- (ii) the vector tangential and normal components of acceleration at time  $t = 2$
- 15 Find the equation to the tangent plane and parametric equations of the normal line to the ellipsoid  $x^2 + y^2 + 4z^2 = 12$  at the point  $(2, 2, 1)$  (5)

## Module IV

*Answer any two questions, each carries 5 marks.*

- 16 Reverse the order of integration and evaluate  $\int_0^1 \int_x^1 \frac{x}{x^2 + y^2} dy dx$  (5)
- 17 If  $R$  is the region bounded by the parabolas  $y = x^2$  and  $y^2 = x$  in the first quadrant, evaluate  $\iint_R (x + y) dA$  (5)
- 18 Use triple integral to find the volume of the solid bounded by the surface  $y = x^2$  and the planes  $y + z = 4, z = 0$ . (5)

## Module V

*Answer any three questions, each carries 5 marks.*

- 19 If  $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$  and  $r = \|\mathbf{r}\|$ , show that  $\nabla \log r = \frac{\mathbf{r}}{r^2}$  (5)
- 20 Examine whether  $\mathbf{F} = (x^2 - yz) \mathbf{i} + (y^2 - zx) \mathbf{j} + (z^2 - xy) \mathbf{k}$  is a conservative field. If so, find the potential function (5)
- 21 Show that  $\nabla^2 f(r) = 2 \frac{f'(r)}{r} + f''(r)$ , where  $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}, r = \|\mathbf{r}\|$  (5)
- 22 Compute the line integral  $\int_C (y^2 dx - x^2 dy)$  along the triangle whose vertices are  $(1, 0), (0, 1)$  and  $(-1, 0)$  (5)
- 23 Show that the line integral  $\int_C (y \sin x dx - \cos x dy)$  is independent of the path and hence evaluate it from  $(0, 1)$  and  $(\pi, -1)$  (5)

## Module VI

*Answer any three questions, each carries 5 marks.*

- 24 Using Green's theorem, find the work done by the force field  $\vec{f}(x, y) = (e^x - y^3) \vec{i} + (\cos y + x^3) \vec{j}$  on a particle that travels once around the unit circle  $x^2 + y^2 = 1$  in the counter clockwise direction. (5)
- 25 Using Green's theorem evaluate  $\int_C (xy + y^2) dx + x^2 dy$ , where  $C$  is the boundary of the area common to the curve  $y = x^2$  and  $y = x$  (5)
- 26 Evaluate the surface integral  $\iint_S xz ds$ , where  $S$  is the part of the plane  $x + y + z = 1$  that lies in the first octant (5)
- 27 Using divergence theorem, evaluate  $\iiint_S \mathbf{F} \cdot \mathbf{n} ds$  where  $\mathbf{F} = (x^2 + y) \mathbf{i} + z^2 \mathbf{j} + (e^y - z) \mathbf{k}$  and  $S$  is the surface of the rectangular solid bounded by the coordinate planes and the planes  $x = 3, y = 1, z = 3$  (5)
- 28 Apply Stokes's theorem to evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$ , where  $\mathbf{F} = (x^2 - y^2) \mathbf{i} + 2xy \mathbf{j}$  and  $C$  is the rectangle in the  $xy$  plane bounded by the lines  $x = 0, y = 0, x = a$  and  $y = b$  (5)

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