

Reg No..... Name:.....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2016
(2015 ADMISSION)

Course Code: MA 101
Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

(Answer all questions. Each question carries 3 marks)

- 1 Find the derivative of $y = (1 + x \cosh^{-1} x)^2$
- 2 Test the convergence of $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$
- 3 Classify the surface $4x^2 + 4y^2 + z^2 + 8y - 4z = 4$
- 4 Convert the rectangular co-ordinate into spherical co-ordinate of $(2, 2\sqrt{3}, -4)$
- 5 Prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ where $f = x^2y$.
- 6 Find the velocity, acceleration and speed of a particle moving along the curve $x = 1 + 3t, y = 3 - 4t, z = 1 + 3t$ at $t = 2$
- 7 Given $z = e^{xy}, x = 2u + v, y = \frac{u}{v}$ Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$
- 8 Find the unit tangent vector and unit normal vector to the curve $x = e^t \cos t, y = e^t \sin t, z = e^t$ at $t = 0$.
- 9 Evaluate $\int_0^{\sqrt{9-y^2}} \int_0^y 2y dx dy$
- 10 Find the area of the region R enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$

(10*3=30 Marks)

PART B

(Answer any 2 questions each question carries 7 marks)

- 11 Find the radius of curvature and interval of curvature of $\sum_{n=1}^{\alpha} \frac{x^n}{2n+3}$
- 12 Test the convergence of $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \dots$
- 13 Determine the Taylor's series expansion of $f(x) = \sin x$ at $x = \pi/4$.

(Answer any 2 questions each question carries 7 marks)

- 14 Find the nature of domain of the following function
1. $f(x, y) = \sqrt{x^2 - y^2}$
 2. $f(x, y) = \ln(x^2 - y)$
- 15 Show that the function $f(x, y) = \frac{x^3 y}{2x^6 + y^2}$ approaches zero as $(x, y) \rightarrow (0, 0)$ along the line $y = mx$.

- 16 Find the trace of the surface $x^2 + y^2 - z^2 = 0$ in the plane $x = 2$ and $y = 1$.
- $$x^2 + y^2 - z^2 = 0$$

(Answer any 2 questions each question carries 7 marks)

- 17 Find the local linear approximation of $f(x, y) = \sqrt{(x^2 + y^2)}$ at $(3, 4)$ and compare the error in approximation by $L(3.04, 3.98)$ with the distance between the points.
- 18 Find the relative extrema of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$
- 19 If $z = e^{xy}$, $x = 2u + v$, $y = \frac{u}{v}$ Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$

(Answer any 2 questions each question carries 7 marks)

- 20 If $r(t) = e^t i + e^{-2t} j + tk$
- 1) Find the scalar tangential and normal component of acceleration at $t = 0$
 - 2) Find the vector tangential and normal component of acceleration at $t = 0$.
- 21 Find the equation of the tangent plane and parametric equations of the normal

line to the surface $z = 4x^3y^2 + 2y - 2$ at the point $P(1, -2, 10)$.

- 22 Find the directional derivative of $f = x^2y - yz^3 + z$ at $(1, -2, 0)$ in the direction of $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$

(Answer any 2 questions each question carries 7 marks)

- 23 Evaluate $\iint_R y \, dA$ where R is the region in the first quadrant enclosed between the

circle $x^2 + y^2 = 25$ and the line $x+y=5$

- 24 Change the order of integration and evaluate $\int_1^2 \int_y^{y^2} y^2 \, dx \, dy$

- 25 Find the volume bounded by the cylinder $x^2 + y^2 = 4$ the planes $y + z = 3$ and $z = 0$.