

Reg. No.:.....

Name.....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, DEC 2016
(2016 ADMISSION)
Course Code: MA 101
Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A*Answer ALL questions*

- 1 (a) Determine whether the series $\sum_{k=1}^{\infty} \left(\frac{3}{4}\right)^{k+2}$ converges and if so, find its (2)
sum.
- (b) Find the Maclaurin series for the function xe^x (3)
- 2 (a) If $z = x^y$, then find $\frac{\partial^2 z}{\partial x \partial y}$ (2)
- (b) Compute the differential dz of the function $z = \tan^{-1}(xy)$. (3)
- 3 (a) Find the domain of $r(t) = \langle \sqrt{5t+1}, t^2 \rangle$, $t_0 = 1$ and $r(t_0)$ (2)
- (b) Find the directional derivative of $f(x, y) = e^{2xy}$ at $P(5, 0)$, in the (3)
direction of $u = -\frac{3}{5}i + \frac{4}{5}j$
- 4 (a) Evaluate $\int_0^1 \int_0^1 \frac{dx dy}{\sqrt{1-x^2} \sqrt{1-y^2}}$ (2)
- (b) Use double integration to find the area of the plane region enclosed by the (3)
given curves $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq \frac{\pi}{4}$
- 5 (a) Confirm that $\phi(x, y, z) = x^2 - 3y^2 + 4z^3$ is a potential function for (2)
 $F(x, y, z) = 2xi - 6yj + 12z^2k$.
- (b) Evaluate $\int_C F \cdot dr$ where $F(x, y) = \sin x i + \cos x j$ where C is the curve (3)
 $r(t) = \pi i + tj$, $0 \leq t \leq 2$
- 6 (a) Using Green's theorem evaluate $\oint_C y dx + x dy$, where C is the unit (2)
circle oriented counter clockwise.
- (b) If σ is any closed surface enclosing a volume V and $F = 2xi + 2yj + (3) 3zk$, Using Divergence theorem show that $\int_{\sigma} F \cdot n dS = 7V$

PART B

*(Each question carries 5 Marks)**Answer any TWO questions*

- 7 Test the nature of the series $\sum_{k=1}^{\infty} \frac{4k^3 - 6k + 5}{8k^7 + k - 8}$
- 8 Check whether the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^k}{k!}$ is absolutely convergent or not.
- 9 Find the radius of convergence and interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{5^k}$

Answer any TWO questions

- 10 If $u = f(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
- 11 A function $f(x, y) = x^2 + y^2$; is given with a local linear approximation $L(x, y) = 2x + 4y - 5$ to $f(x, y)$ at a point P. Determine the point P.
- 12 Find the absolute extrema of the function $f(x, y) = xy - 4x$ on R where R is the triangular region with vertices (0,0) (0,4) and (4,0).

Answer any TWO questions

- 13 Evaluate the definite integral $\int_0^1 (e^{2t}i + e^{-t}j + 2\sqrt{t}k)dt$.
- 14 Find the velocity, acceleration, speed, scalar tangential and normal components of acceleration at the given t of $r(t) = 3 \sin t i + 2 \cos t j - \sin 2t k$; $t = \frac{\pi}{2}$
- 15 Find the equation of the tangent plane and parametric equation for the normal line to the surface $z = 4x^3y^2 + 2y - 2$ at the point (1,-2,10)

Answer any TWO questions

- 16 Evaluate the integral $\int_0^4 \int_y^4 \frac{x}{x^2 + y^2} dx dy$ by first reversing the order of integration.
- 17 Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x dz dx dy$
- 18 Find the volume of the solid in the first octant bounded by the co-ordinate

planes and the plane $x + y + z = 1$

PART C

(Each question carries 5 Marks)

Answer any THREE questions

- 19 Find div F and curl F of $F(x, y, z) = x^2yi + 2y^3zj + 3zk$
- 20 Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r = \|xi + yj + zk\|$
- 21 Find the work done by the force field
 $F(x, y, z) = (x^2 + xy) i + (y - x^2y)j$ on a particle that moves along the curve $C: x = t, y = \frac{1}{t}, 1 \leq t \leq 3$
- 22 Evaluate $\int F \cdot dr$ where $F(x, y) = y i - x j$ along the triangle joining the vertices (0,0), (1,0), and (0,1).
- 23 Determine whether $F(x, y) = 4y i + 4xj$ is a conservative vector field. If so, find the potential function and the potential energy.

Answer any THREE questions

- 24 Using Green's theorem evaluate $\oint_C (e^x + y^2) dx + (e^y + x^2) dy$ where C is the boundary of the region between $y = x^2$ and $y = 2x$.
- 25 Evaluate the surface integral $\iint_{\sigma} \frac{x^2+y^2}{y} dS$ over the surface σ represented by the vector valued function
 $r(u, v) = 2\cos v i + u j + 2\sin v k, 1 \leq u \leq 3, 0 \leq v \leq \pi$
- 26 Using Divergence Theorem evaluate $\iint_{\sigma} F \cdot n ds$ where $F(x, y, z) = (x - z)i + (y - x)j + (2z - y)k$, σ is the surface of the cylindrical solid bounded by $x^2 + y^2 = a^2, z = 0, z = 1$.
- 27 Determine whether the vector field $F(x, y, z) = 4(x^3 - x)i + 4(y^3 - y)j + 4(z^3 - z)k$ is free of sources and sinks. If it is not, locate them.
- 28 Using Stokes theorem evaluate $\int_C F \cdot dr$ where
 $F(x, y, z) = x^2i + 4xy^3j + y^2xk$,
 C is the rectangle: $0 \leq x \leq 1, 0 \leq y \leq 3$ in the plane $z = y$