**A**                                   **A7001**

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### APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

**Course Code: MA101**

**Course Name: CALCULUS**

Max. Marks: 100                                                        Duration: 3 Hours

## PART A
*Answer all questions, each carries 5 marks.*                          Marks

1  a) Test the convergence of the series $\sum_{k=1}^{\infty} \dfrac{1}{\sqrt[3]{2k-1}}$ .                          (2)

   b) Find the radius of convergence of $\sum_{n=1}^{\infty} \dfrac{x^n}{2n+3}$.                          (3)

2  a) Find the Slope of the surface $z = xe^{-y} + 5y$ in the y-direction at the point (4,0).     (2)

   b) Find the derivative of $z = \sqrt{1+x-2xy^4}$ with respect to $t$ along the path $x = \log t,\, y = 2t$.                          (3)

3  a) Find the directional derivative of $f = x^2 y - yz^3 + z$ at $(-1,2,0)$ in the direction of $a = 2i + j + 2k$.                          (2)

   b) Find the unit tangent vector and unit normal vector to $r(t) = 4\cos t\, i + 4\sin t\, j + t k$ at $t = \dfrac{\pi}{2}$.                          (3)

4  a) Evaluate $\int_{0}^{\log 3} \int_{0}^{\log 2} e^{x+2y}\, dy\, dx$.                          (2)

   b) Evaluate $\iint_R xy\, dA$, where R is the region bounded by the curves $y = x^2$ and $x = y^2$.                          (3)

5  (a) Find the divergence and curl of the vector $F(x,y,z) = yz\,i + xy^2\,j + yz^2\,k$.     (2)

   (b) Evaluate $\int_C (3x^2 + y^2)dx + 2xy\, dy$ along the circular arc $C$ given by $x = \cos t,\, y = \sin t$ for $0 \le t \le \dfrac{\pi}{2}$.                          (3)

6  (a) Use line integral to evaluate the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.     (2)

   (b) Evaluate $\int_C (x^2 - 3y)dx + 3x\, dy$, where $C$ is the circle $x^2 + y^2 = 4$.     (3)

## PART B
### Module 1
*Answer any two questions, each carries 5 marks.*

7  Test the convergence or divergence of the series $\sum_{n=1}^{\infty} \left(\dfrac{n}{n+1}\right)^{n^2}$ .     (5)

8    Test the absolute convergence of $\sum\limits_{k=1}^{\infty}(-1)^{k+1}\dfrac{(2k)!}{(3k-2)!}$.    (5)

9    Find the Taylor series for $\dfrac{1}{1+x}$ at $x=2$.    (5)

## Module 1I
### Answer any two questions, each carries 5 marks.

10    Find the local linear approximation L to $f(x,y)=\log(xy)$ at P(1,2) and compare the error in approximating f by L at Q(1.01, 2.01) with the distance between P and Q.    (5)

11    Let $w=4x^2+4y^2+z^2, x=\rho\sin\phi\cos\theta, y=\rho\sin\phi\sin\theta, z=\rho\cos\phi.$ Find $\dfrac{\partial w}{\partial \rho}, \dfrac{\partial w}{\partial \phi}$ and $\dfrac{\partial w}{\partial \theta}$.    (5)

12    Locate all relative extrema and saddle points of $f(x,y)=4xy-x^4-y^4$.    (5)

## Module 1II
### Answer any two questions, each carries 5 marks.

13    Find the equation of the tangent plane and parametric equation for the normal line to the surface $x^2+y^2+z^2=25$ at the point $(3,0,4)$.    (5)

14    A particle is moving along the curve $r(t)=(t^3-2t)i+(t^2-4)j$ where $t$ denotes the time. Find the scalar tangential and normal components of acceleration at $t=1$. Also find the vector tangential and normal components of acceleration at $t=1$.    (5)

15    The graphs of $r_1(t)=t^2i+tj+3t^3k$ and $r_2(t)=(t-1)i+\dfrac{1}{4}t^2j+(5-t)k$ are intersect at the point $P(1,1,3)$. Find, to the nearest degree, the acute angle between the tangent lines to the graphs of $r_1(t)\,\&\,r_2(t)$ at the point $P(1,1,3)$.    (5)

## Module 1V
### Answer any two questions, each carries5 marks.

16    Change the order of integration and evaluate $\int\limits_0^1\int\limits_{4x}^4 e^{-y^2}\,dydx$.    (5)

17    Use triple integral to find the volume bounded by the cylinder $x^2+y^2=9$ and between the planes $z=1$ and $x+z=5$.    (5)

18    Find the area of the region enclosed between the parabola $y=\dfrac{x^2}{2}$ and the line $y=2x$.    (5)

## Module V
### Answer any three questions, each carries5 marks.

19    Determine whether $F(x,y)=(\cos y+y\cos x)i+(\sin x-x\sin y)j$ is a conservative vector field. If so find the potential function for it.    (5)

20    Show that the integral $\int\limits_{(1,1)}^{(3,3)}(e^x\log y-\dfrac{e^y}{x})dx+(\dfrac{e^x}{y}-e^y\log x)dy$ ,where $x$ and $y$ are positive is independent of the path and find its value.    (5)

21    Find the work done by the force field $F(x,y,z)=xyi+yzj+xzk$ on a particle that moves along the curve $C:r(t)=ti+t^2j+t^3k\,(0\le t\le 1)$.    (5)

22    Let $\bar{r} = xi + yj + zk$ and $r =\| \bar{r} \|$, let $f$ be a differentiable function of one variable,    (5)
then show that $\nabla f(r) = \dfrac{f'(r)}{r}\bar{r}.$

23    Find $\nabla.(\nabla \times F)$ and $\nabla \times (\nabla \times F)$ where $F(x,y,z) = e^{xz}i + 4xe^{y}j - e^{yz}k.$    (5)

## Module VI
### *Answer any three questions, each carries 5 marks.*

24    Use Green's Theorem to evaluate $\displaystyle\int_{C} \log(1+y)dx - \frac{xy}{(1+y)}dy$, where $C$ is the    (5)
triangle with vertices (0,0), (2,0) and (0,4).

25    Evaluate the surface integral $\displaystyle\iint_{\sigma} xz\,ds$, where $\sigma$ is the part of the plane $x + y + z = 1$    (5)
that lies in the first octant.

26    Using Stoke's Theorem evaluate $\displaystyle\int_{C} F.dr$ where $F(x,y,z) = xzi + 4x^2y^2j + yxk, C$    (5)
is the rectangle $0 \le x \le 1, 0 \le y \le 3$ in the plane $z = y.$

27    Using        Divergence        Theorem        evaluate        $\displaystyle\iint_{\sigma} \bar{F}.n\,ds$        where    (5)

$F(x,y,z) = x^3 i + y^3 j + z^3 k, \sigma$ is the surface of the cylindrical solid bounded by
$x^2 + y^2 = 4, z = 0$ and $z = 4.$

28    Determine whether the vector fields are free of sources and sinks. If it is not,    (5)
locate them
(i) $(y+z)i - xz^3 j + x^2 \sin y\, k$    (ii) $xy\,i - 2xy\, j + y^2 k$
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