

Reg. No. _____ Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, JUNE/JULY 2017

Course Code: **MA 101**
Course Name: **CALCULUS**
(For 2015 Admission and 2016 Admission)

Max. Marks :100

Duration: 3 hours

PART A

Answer all questions. Each question carries 5 marks.

1. (a) Find the interval of convergence and radius of convergence of the infinite series $\sum_{n=0}^{\infty} n! x^n$ (2)
- (b) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{5n-1}$ converges or not (3)
2. (a) Find the slope of the surface $z = \sqrt{3x + 2y}$ in the y-direction at the point (4,2) (2)
- (b) Find the derivative of $w = x^2 + y^2$ with respect to t along the path $x = at^2, y = 2at$ (3)
3. (a) Find the directional derivative of $f(x, y) = xe^y$ at (1,1) in the direction of the vector $i - j$ (2)
- (b) If $\vec{F}(t)$ has a constant direction, then prove that $\vec{F} \times \frac{d\vec{F}}{dt} = 0$ (3)
4. (a) Evaluate $\int_0^1 \int_0^1 \frac{1}{\sqrt{(1-x^2)(1-y^2)}} dx dy$ (2)
- (b) Evaluate $\iint_R \frac{\sin x}{x} dx dy$ where R is the triangular region bounded by the x-axis, $y = x$ and $x = 1$. (3)
5. (a) Show that $\int_A^B (2xy + z^3) dx + x^2 dy + 3xz^2 dz$ is independent of the path joining the points A and B. (2)
- (b) If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ (3)
6. (a) Using line integral evaluate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (2)
- (b) Evaluate $\int_C (e^x dx + 2y dy - dz)$ where C is the curve $x^2 + y^2 = 4, z = 2$. (3)

PART B

Answer any two questions each Module I to IV

Module I

7. Determine whether the series converge or diverge $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ (5)
8. Check the absolute convergence or divergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!}{3^n}$ (5)

9. Find the Taylor series expansion of $\log \cos x$ about the point $\frac{\pi}{3}$ (5)

Module II

10. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, Show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$ (5)
11. The length, width and height of a rectangular box are measured with an error of at most 5%. Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box. (5)
12. Locate all relative extrema and saddle points of $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ (5)

Module III

13. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$ (5)
14. Let $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then prove that $\nabla f(r) = \frac{f'(r)}{r} \vec{r}$. (5)
15. Find an equation of the tangent plane to the ellipsoid $2x^2 + 3y^2 + z^2 = 9$ at the point $(2, 1, 1)$ and determine the acute angle that this plane makes with the XY plane. (5)

Module IV

16. Change the order of integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ (5)
17. Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} y(x^2 + y^2) \, dx \, dy$ using polar co-ordinates (5)
18. Find the volume of the paraboloid of revolution $x^2 + y^2 = 4z$ cut off by the plane $z = 4$ (5)

Module V

Answer any 3 questions.

19. Evaluate the line integral $\int_C (xy + z^3) \, ds$ from $(1, 0, 0)$ to $(-1, 0, \pi)$ along the helix C that is represented by the parametric equations $x = \cos t, y = \sin t, z = t$ (5)
20. Evaluate the line integral $\int_C (y - x) \, dx + x^2 y \, dy$ along the curve $C, y^2 = x^3$ from $(1, -1)$ to $(1, 1)$ (5)
21. Find the work done by the force field $\vec{F} = (x + y)i + xyj - z^2k$ along the line segment from $(0, 0, 0)$ to $(1, 3, 1)$ and then to $(2, -1, 5)$. (5)
22. Show that $\vec{F} = (2xy + z^3)i + x^2j + 3xz^2k$ is a conservative vector field. Also find its scalar potential. (5)
23. Find the values of constants a, b, c so that $\vec{F} = (axy + bz^3)i + (3x^2 - cz)j + (3xz^2 - y)k$ may be irrotational. For these values of a, b, c find the scalar potential of \vec{F} (5)

Module VI

Answer any 3 questions.

24. Verify Green's theorem for $\int_C (xy + y^2)dx + x^2 dy$ where C is bounded by $y = x$ and $y = x^2$ (5)
25. Apply Green's theorem to evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$ (5)
26. Apply Stokes theorem to evaluate $\int_C (x + y)dx + (2x - y)dy + (y + z)dz$ where C is the boundary of the triangle with vertices (0,0,0), (2,0,0) and (0,3,0) (5)
27. Use Divergence theorem to evaluate $\iint_S \vec{F} \cdot \vec{n}dS$ where $\vec{F} = xi + zj + yzk$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. Also verify this result by computing the surface integral over S (5)
28. State Divergence theorem. Also evaluate $\iint_S \vec{F} \cdot \vec{n}dS$ where $\vec{F} = axi + byj + czk$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ (5)
