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Reg. No.________________ Name: ____________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE SPECIAL EXAMINATION, AUGUST 2016

Course Code: MA-102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 hrs

PART A

Answer all questions Each carries 3 marks

1. Find ordinary differential equation for the basis $e^{-x\sqrt{2}}$, $xe^{-x\sqrt{2}}$
2. Reduce $y'' = y'$ to 1^{st} order differential equation and solve.
3. Find the particular solution to $(D^4 - m^4)\ y = \sin\ mx$
4. Use variation of parameters to solve $y'' + y = \sec x$
5. Find the Fourier coefficient a_n for the function $f(x) = 1 + |x|$ defined in $-3 < x < 3$
6. Develop the Fourier Sine series of $f(x) = x$ in $0 < x < \pi$
7. Obtain the partial differential equation by eliminating arbitrary function from $x^2 + y^2 + z^2 = f(xy)$
8. Solve $y^2zp + x^2zq = xy^2$
9. Solve $u_x + u_y = 0$ using method of separation of variables
10. A finite string of length L is fixed at both ends and is released from rest with a displacement $f(x)$. What are the initial and boundary conditions involved in this problem?
11. Write all the possible solutions of one-dimensional heat transfer equation
12. Find the steady state temperature distribution in a rod of length 30cm having the ends at 20^0 C and 80^0 C respectively.

PART B

Answer one full question from each module

Module -I

13. (a)Verify linear independence of $e^{-x}cosx$ and $e^{-x}sinx$ using Wronskian and hence solve the initial value problem $y'' + 2y' + 2y = 0,\ y(0) = 0,\ y'(0) = 15$

(b) Find the general solution of the equation $x^2y'' + xy' + (x^2 - 0.25)y = 0$ If $y_1 = \frac{\cos x}{\sqrt{x}}$

OR

14. (a) Find a second order homogeneous linear ODE for which x, $x\log x$ are solutions and solve the IVP with $y(1) = 2$, $y'(1) = 4$.

(b) Solve the IVP $y'' - 4y' + 9y = 0$, $y(0) = 0, y'(0) = -8$

Module- II

15. (a) Solve $(D^2 - 2D + 5)y = e^{2x}\sin x$

(b) Solve $((x+1)^2\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} = (2x+3)(2x+4)$

OR

16. (a) Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

(b)Solve $y'' - 4y' + 5y = \frac{e^{2x}}{\sin x}$ by method of variation of parameters

Module - III

17. (a)Find the Fourier series representation of $f(x) = x\sin x$ periodic with period 2π, defined in $0 < x < 2\pi$

(b)Find the Fourier cosine series of $f(x) = \cos x$, $0 < x < \pi/2$

OR

18. (a)Find the Fourier series expansion of $f(x) = e^{-x}$ in $-c < x < c$

(b)Develop the Sine series representation of $f(x) = \begin{cases} x, & 0 < x < 2 \\ 4 - x, & 2 < x < 4 \end{cases}$

Module - IV

19. (a) Solve $(y + zx)p - (x + yz)q = x^2 - y^2$

(b)Find the differential equation of all spheres of fixed radius having their centres in XY -plane.

OR

20. (a)Solve $\left(D^2 - 2DD' - 15D'^2\right)z = 12xy$

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(b)Find the particular integral of

$$\left(D^3 - 7DD'^2 - 6D'^3\right)z = \sin(x + 2y) + e^{3x+y}$$

Module – V

21. A tightly stretched homogeneous string of length 20cm with its fixed ends executes transverse vibrations. Motion starts with zero initial velocity by displacing the string into the form $f(x) = K(x^2 - x^3)$ .Find the deflection $u(x, t)$ at any time t.

OR

22. A tightly stretched string of length 'a' with fixed ends is initially in equilibrium position. Find the displacement $u(x, t)$ of the string if it is set vibrating by giving each of its points a velocity $v_0 sin^3\left(\frac{\pi x}{a}\right)$.

Module - VI

23. Find the temperature distribution in a rod of length 2m whose end points maintained at temperature zero and the initial temperature $f(x) = 100(2x - x^2)$

OR

24. The temperatures at the ends of a bar of length l cm with insulated sides are $30^o C$ and $90^o C$ respectively until steady state conditions prevail. If the temperature at each end is then suddenly reduced to $0^o C$ and maintained so, find the temperature distribution at a distance x at time t.