Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1 Solve the initial value problem $xy' = y - 1, y(0) = 1$ (3)

2 Solve the following differential equation by reducing it to first order $xy'' = 2y'$. (3)

3 Find the particular integral of $(D^2 + 3D + 2)y = 3$. (3)

4 Find the particular integral of $y'' + y = \sin x$. (3)

5 Obtain the Fourier series expansion for the function $f(x) = x$ in the range $-\pi < x < \pi$. (3)

6 Find the Fourier sine series of the function $f(x) = \pi x - x^2$ in the interval $(0, \pi)$ (3)

7 Form a partial differential equation by eliminating the arbitrary function in $xyz = \emptyset(x + y + z)$ (3)

8 Solve $r + s - 2t = e^{x+y}$. (3)

9 Solve one dimensional wave equation for $k < 0$. (3)

10 Solve $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} - u = 0, u(x, 0) = 6e^{-3x}$ using method of separation of variables. (3)

11 Find the steady state temperature distribution in a rod of length 30cm if the ends are kept at 20^0C and 80^0C. (3)

12 Write down the possible solutions of one dimensional heat equation. (3)

PART B

Answer six questions, one full question from each module.

Module I

13 a) Verify that the given functions $x^{\frac{3}{2}}, x^{-\frac{1}{2}}$ are linearly independent and form a basis of solution space of given ODE $4x^2y'' - 3y = 0$. (6)

b) Solve the boundary value problem: (5)

$$y'' - 10y' + 25y = 0, \quad y(0) = 1, \quad y(1) = 0.$$

OR

14 a) Find the general solution of $y'''' + 2y'' + y = 0$. (6)

b) Find a fundamental set of solutions of $2t^2y'' + 3ty' - y = 0, t < 0$. Given that $y_1(t) = \frac{1}{t}$ is a solution. (5)

Module II

15 a) Find the particular integral of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4cos^2x$. (6)

b) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ using method of variation of parameters. (5)

OR

16 a) Solve $x^2y'' - xy' - 3y = x^2 \ln x$ (6)

b) Solve $y'''' - 2y''' + 5y'' - 8y' + 4y = e^x$. (5)

Module III

17 a) Obtain the Fourier series expansion of $f(x) = x \sin x$ in the interval $(-\pi, \pi)$. (6)

b) Find the half range sine series of $f(x) = k$ in the interval $(0, \pi)$. (5)

OR

18 a) Find the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $(0, 2\pi)$. (6)

b) Find the half range sine series of $f(x) = e^x$ in $(0,1)$. (5)

Module IV

19 a) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$ (6)

b) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ (5)

OR

20 a) Form the PDE by eliminating a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (6)

b) Solve $(x + y)zp + (x - y)zq = x^2 + y^2$. (5)

Module V

21 A tightly stretched violin string of length 'a' and fixed at both ends is plucked at its mid-point and assumes initially the shape of a triangle of height 'h'. Find the displacement u(x,t) at any distance 'x' and any time 't' after the string is released from rest. (10)

OR

22 Solve the PDE $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (10)

Boundary conditions are $u(0,t) = u(l,t) = 0,\ t \geq 0$

Initial conditions are $y(x,0) = a \sin\left(\frac{\pi x}{l}\right)$ and $\frac{\partial y}{\partial t} = 0 \quad at \quad t = 0$.

Module VI

23 A rod, 30 cm long has its ends A and B kept at 20^0C and 80^0C respectively, until the steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0C and kept so. Find the resulting temperature function u(x,t) taking x=0 at A. (10)

OR

24 A long iron rod with insulated lateral surface has its left end maintained at a temperature 0^0C and its right end at x=2, maintained at 100^0C. Determine the temperature as a function of 'x' and 't' if the initial temperature is (10)

$$u(x,0) = \begin{cases} 100x, & 0 < x < 1 \\ 100, & 1 < x < 2 \end{cases}.$$

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