

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**SECOND SEMESTER B.TECH DEGREE EXAMINATION, JULY 2017**

**Course Code: MA102**

**Course Name: DIFFERENTIAL EQUATIONS**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions. Each carries 3 marks.*

- 1 Find a second order homogeneous linear ODE for which  $e^{-x}$  and  $e^{-2x}$  are the solutions. (3)
- 2 Find a basis of solutions of  $y^{11} - y^1 = 0$ . (3)
- 3 Find the particular integral of  $(D^2 - 4)y = x^2$ . (3)
- 4 Solve  $(D^2 + 3D + 2)y = 5$ . (3)
- 5 Expand  $\pi x - x^2$  in a half range sine series in the interval  $(0, \pi)$ . (3)
- 6 Expand  $f(x)$  in Fourier series in the interval  $(-2, 2)$  when  

$$f(x) = \begin{cases} 0 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$$
 (3)
- 7 Obtain the partial differential equation by eliminating the arbitrary function from  $z = f(x^2 + y^2)$ . (3)
- 8 Solve  $xp + yq = 3z$ . (3)
- 9 Using the method of separation of variables solve  $u_{xy} - u = 0$ . (3)
- 10 Write down the possible solutions of one dimensional wave equation. (3)
- 11 Find the solution of one dimensional heat equation in steady state condition. (3)
- 12 State one dimensional heat equation with boundary conditions and initial conditions for solving it. (3)

**PART B**

*Answer six questions, one full question from each module.*

**Module 1**

- 13 a) Reduce to first order and solve  $x^2y^{11} - 5xy^1 + 9y = 0$ . Given  $y_1 = x^3$  is a solution. (6)
- b) Solve the initial value problem  $4y^{11} - 25y = 0$  where  $y(0) = 0$ ,  $y^1(0) = -5$ . (5)

**OR**

- 14 a) Show that the functions  $e^{-x}\text{Cos}x$  and  $e^{-x}\text{Sin}x$  are linearly independent. Form a second order linear ODE having these functions as solutions. (6)
- b) Solve  $y^{1V} - 2y^{111} + 5y^{11} - 8y^1 + 4y = 0$ . (5)

**Module 1I**

- 15 a) Solve  $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left(x + \frac{1}{x}\right)$ . (6)
- b) Solve  $y^{11} - 4y^1 + 3y = e^x \text{Cos } 2x$ . (5)

**OR**

- 16 a) Solve  $y^{11} + y = \text{Cosec } x$  using the method of variation of parameters. (6)
- b) Solve  $(D^2 - 2D + 1)y = x \text{Sin}x$ . (5)

**Module III**

- 17 a) If  $f(x) = x + x^2$  for  $-\pi < x < \pi$  find the Fourier series expansion of  $f(x)$ . (6)  
 b) Express  $f(x) = |x|$   $-\pi < x < \pi$  as Fourier series. (5)

**OR**

- 18 a) Obtain Fourier series for the function  $f(x) = \begin{cases} \pi x & \text{when } 0 \leq x \leq 1 \\ \pi(2-x) & \text{when } 1 \leq x \leq 2 \end{cases}$  (6)  
 b) Obtain the half range cosine series for  $f(x) = x$  in the 2interval  $0 \leq x \leq \pi$ . (5)  
 Hence show that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ .

**Module IV**

- 19 a) Solve  $xp - yq = y^2 - x^2$ . (6)  
 b) Solve  $\frac{\partial^2 z}{\partial x^2} - 7 \frac{\partial^2 z}{\partial x \partial y} + 12 \frac{\partial^2 z}{\partial y^2} = e^{x-y}$ . (5)

**OR**

- 20 a) Solve  $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \text{Sin}x \text{Cos}2y$ . (6)  
 b) Solve  $p - 2q = 3x^2 \text{Sin}(y + 2x)$ . (5)

**Module V**

- 21 Derive one dimensional wave equation. (10)

**OR**

- 22 a) A tightly stretched homogeneous string of length  $l$  with its fixed ends at  $x = 0$  and  $x = l$  executes transverse vibrations. Motion starts with zero initial velocity by displacing the string into the form  $f(x) = k(x^2 - x^3)$ . Find the deflection  $u(x, t)$  at any time  $t$ . (10)

**Module VI**

- 23 Find the temperature distribution in a rod of length  $2m$  whose end points are maintained at temperature zero and the initial temperature is  $f(x) = 100(2x - x^2)$ . (10)

**OR**

- 24 A long iron rod with insulated lateral surface has its left end maintained at a temperature  $0^\circ\text{C}$  and its right end at  $x=2$  maintained at  $100^\circ\text{C}$ . Determine the temperature as a function of  $x$  and  $t$  if the initial temperature is (10)

$$u(x, 0) = \begin{cases} 100x & 0 < x < 1 \\ 100 & 1 < x < 2 \end{cases}$$

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