Reg. No.____________ Name:______________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

MA 102: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3Hours

PART A

Answer all questions. 3 marks each.

1. Solve the initial value problem $y^{II} - y = 0,\ y(0) = 4, y^{I}(0) = -2$
2. Show that $e^{2x},\ \ e^{3x}$are linearly independent solutions of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0\ \ in\ -\infty < x < +\infty$.What is its general solution?
3. Solve $\frac{d^3y}{dx^3}$ -4$\frac{d^2y}{dx^2} + 5\frac{dy}{dx}$ -2y =0
4. Find the particular integral of $(D^2 + 4D + 1)y = e^x sin3x$
5. Find the Fourier series of f(x)=x , $-\pi\ \le x \le \pi$
6. Obtain the half range cosine series of f(x)= x^2 , $0 \le x \le C$
7. Form the partial differential equation from $z = xg(y) + yf(x)$
8. Solve $(y - z)p + (x - y)q = (z - x)$
9. Write down the important assumption when derive one dimensional wave equation.
10. Solve $3u_x$ $+2u_y$=0 with u(x,0)=4e^{-x} by the method of separation of variables.
11. Solve one dimensional heat equation when k> 0
12. Write down the possible solutions of one dimensional heat equation.

PART B

Answer six questions, one full question from each module.

Module I

13. a) Solve the initial value problem $y^{II} - 4y^1 + 13y = 0$ with y(0)= -1 ,$y^1(0) = 2$ **(6)**

    b) Solve the boundary value problem$y^{II} - 10y^{I} + 25y = 0$, y(0)=1,y(1)=0 **(5)**

OR

14. a) Show that $y_1(x) = e^{-4x}$ and $y_2(x) = xe^{-4x}$ are solutions of the differential equation $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$. Are they linearly independent? **(6)**

    b) Find the general solution of $(D^4 + 3D^2 - 4)y = 0$. **(5)**

Module II

15. a) Solve $(D^3 + 8\,)y = sinx \cos x + e^{-2x}$ **(6)**

    b) Solve $y^{II} + y = \tan x$ by the method of variation of parameters. **(5)**

OR

16. a) Solve $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d2y}{dx2} + 2y = \frac{1}{x}$ **(6)**

b) Solve $(D^2 + 2D - 3)y = e^x\, cosx$ **(5)**

Module III

17. a) Find the Fourier series of $f(x) = \begin{cases} -1 + x, -\pi < x < 0 \\ 1 + x,\ 0 < x < \pi \end{cases}$ **(6)**

b) Find the half range sine series of f$(x) = \begin{cases} x,\ 0 < x < 1 \\ 2 - x,\ 1 < x < 2 \end{cases}$ **(5)**

OR

18. a) Obtain the Fourier series of $f(x) = \begin{cases} -\frac{\pi}{4}, -\pi < x < 0 \\ \pi/4,\ 0 < x < \pi \end{cases}$ **(6)**

b) Find the half range cosine series of $f(x) = x, 0 < x < l$ **(5)**

Module IV

19. a) Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3$ **(6)**

b) Find the Particular Integral of $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x^2 \partial y} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x + 2y)$ **(5)**

OR

20. a) Solve $(D^2 + DD' - 6D'^2)z = y\, sinx$ **(6)**

b) Solve $(m\,z - ny)p + (nx - lz)q = ly - m\,x$ **(5)**

Module V

21. Solve the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(0,t) = 0, u(l,t) = 0\ for$ all t and initial conditions $u(x,0) = f(x), \left.\frac{\partial u}{\partial t}\right\}_{t=0} = g(x)$. **(10)**

OR

22. A sting of length 20cm fixed at both ends is displaced from its position of equilibrium, by each of its points an initial velocity given by $= \begin{cases} x, & 0 < x \le 10 \\ 20 - x, & 10 \le x \le 20 \end{cases}$, x being the distance from one end. Determine the displacement at any subsequent time. **(10)**

Module VI

23. Derive one-dimensional heat equation. **(10)**

OR

24. Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0°C, assuming that the initial temperature is $f(x) = \begin{cases} x, & 0 < x < L/2 \\ L - x, & L/2 < x < L \end{cases}$ **(10)**

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