

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
SECOND SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

- 1 Find a general solution of the ordinary differential equation $y'' + y = 0$ (3)
- 2 Find the Wronskian of $e^x \cos 2x$ and $e^x \sin 2x$ (3)
- 3 Find the particular integral of the differential equation $y'' + y = \cosh 5x$ (3)
- 4 Using a suitable transformation, convert the differential equation.
 $(3x + 2)^2 y'' + 5(3x + 2)y' - 3y = x^2 + x + 1$ into a linear differential (3)
equation with constant coefficients.
- 5 If $f(x)$ is a periodic function of period $2L$ defined in $[-L, L]$. Write down Euler's (3)
Formulas a_0, a_n, b_n for $f(x)$.
- 6 Find the Fourier cosine series of $f(x) = x^2$ in $0 < x \leq c$. (3)
- 7 Find the partial differential equation of all spheres of fixed radius having their (3)
centres in xy -plane.
- 8 Find the particular integral of $r + s - 2t = \sqrt{2x + y}$. (3)
- 9 Write any three assumptions involved in the derivation of one dimensional wave (3)
equation.
- 10 Solve $x \frac{\partial u}{\partial x} - 2y \frac{\partial u}{\partial y} = 0$ using method of separation of variables. (3)
- 11 Find the steady state temperature distribution in a rod of 30 cm having its ends at (3)
 20°C and 80°C respectively.
- 12 Write down the possible solutions of the one dimensional heat equation. (3)

PART B

Answer six questions, one full question from each module

Module 1

- 13 a) Solve the initial value problem $y'' + 4y' + 5y = 0, y(0) = 2, y'(0) = -5$. (5)
- b) Find a basis of solutions of the ODE $(x^2 - x)y'' - xy' + y = 0$, if $y_1 = x$ is a (6)

solution.

OR

14 a) Reduce to first order and solve $y'' + (1 + \frac{1}{y})(y')^2 = 0$ (5)

b) Solve the initial value problem $9y'' - 30y' + 25y = 0, y(0) = 3, y'(0) = 10$. (6)

Module II

15 a) Solve $y'' - 2y' + 5y = e^{2x} \sin x$. (5)

b) Using method variation of parameters solve $y'' + 4y = \tan 2x$ (6)

OR

16 a) Solve $x^3 y''' + 3x^2 y'' + xy' + y = x + \log x$ (5)

b) Solve using method of variation of parameters $y'' - 2y' + y = \frac{e^x}{x}$ (6)

Module III

17 Find the Fourier series of periodic function $f(x) = \begin{cases} -x, & -1 \leq x \leq 0 \\ x, & 0 \leq x \leq 1 \end{cases}$ with period π (11)

2. Hence prove that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.

OR

18 Find the Fourier series of periodic function $f(x) = x \sin x, 0 < x < 2\pi$ with period 2π . (11)

Module IV

19 a) Solve $p - 2q = 3x^2 \sin(y+2x)$. (5)

b) Solve $r + s - 6t = y \sin x$. (6)

OR

20 a) Solve $x(y - z)p + y(z - x)q = z(x - y)$. (5)

b) Solve $(D^2 - 2DD' - 15D'^2)z = 12xy$. (6)

Module V

21 A tightly stretched string of length L is fixed at both ends. Find the displacement $u(x,t)$ if the string is given an initial displacement $f(x)$ and an initial velocity $g(x)$. (10)

OR

22 A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $u = v_0 \sin^3\left(\frac{\pi x}{l}\right), 0 \leq x \leq l$. If it is released from rest from this position, find the displacement function $u(x,t)$ (10)

Module VI

- 23 The ends A and B of a rod of length L are maintained at temperatures 0°C and 100°C respectively until steady state conditions prevail. Suddenly the temperature at the end A is increased to 20°C and the end B is decreased to 60°C . Find the temperature distribution in the rod at time t. (10)

OR

- 24 Find the temperature distribution in a rod of length 2 m whose end points are maintained at temperature zero and the initial temperature is $f(x) = 100(2x - x^2)$, $0 \leq x \leq 2$. (10)
