A   R3902

Reg No.:_____   Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018
Course Code: MA201
Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100   Duration: 3 Hours

PART A
Answer any two full questions, each carries 15 marks   Marks

1 a) Prove that $f(z) = e^{x+iy}$ is analytic. Find $f'(z)$.   (7)

   b) Show that $v = 3x^2y - y^3$ is harmonic. Also find the harmonic conjugate of v.   (8)

2 a) Find the linear fractional transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ (8)
onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively.

   b) Find the image of the lines $x = a$ and $y = b$ where a and b are constants, under   (7)
the transformation $w = z^2$

3 a) If $f(z) = u + iv$ is analytic, prove that $u = c_1$ and $v = c_2$ are families of curves   (7)
cutting orthogonally.

   b) Prove that $w = \frac{z-i}{1-iz}$ maps the upper half plane (y>0) into the interior of $|w| = 1$ (8)

PART B
Answer any two full questions, each carries 15 marks

4 a) Expand $f(z) = \frac{1}{z^3}$ as Taylor's series about $z = 2$ (7)

   b) Evaluate $\oint_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$ where C is $|z| = 3$, using Cauchy's integral formula.   (8)

5 a) Evaluate $\oint_C \frac{z-23}{z^2 - 4z - 5} dz$ where $C: |z - 2 - i| = 3.2$, using Cauchy's residue   (7)
theorem.

   b) Show that $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3} dx = \frac{3\pi}{8}$ (8)

6 a) Find the Laurent's series expansion of $f(z) = \frac{e^{2z}}{(z+1)^3}$ about $z = -1$ (7)

   b) Find the poles and residues of the function $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ (8)

PART C
Answer any two full questions, each carries 20 marks

7 a) Find the Eigen value and Eigen vector of the matrix $A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ (8)

   b) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$ (6)

   c) Solve the system of equations $x - y + z = 0, -x + y - z = 0,$ (6)

   $$10y + 25z = 90 , 20x + 10y = 80$$

8 a) Find out what type of conic section the quadratic form   (8)

   $q = 3x_1^2 + 21x_1x_2 + 3x_2^2 = 0$ represents.

   b) Show that the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & \sqrt{3} \\ 0 & \sqrt{3} & 1 \end{bmatrix}$ is orthogonal.   (6)

   c) Show that the system of equations are inconsistent.   (6)

   $2x + 6y = -11, \qquad 6x + 20y - 6z = -3, \quad 6y - 18z = -1$

9 a) (i) Show that the vectors $(1,-1,0),(1,3,-1)$ and $(5,3,-2)$ are linearly   (8)

   dependent.

   (ii) Show that the quadratic form $4x_1^2 + 12x_1x_2 + 13x_2^2 = 16$ is positive

   definite.

   b) Diagonalize the matrix $A = \begin{bmatrix} 6 & 0 & 0 \\ 12 & 2 & 0 \\ 21 & -6 & 9 \end{bmatrix}$ (12)

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