No. of Pages: **B**

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2018

Branch: Civil Engineering

Stream: Structural Engineering

01CE6103 Theory of Elasticity

Answer *any two full* questions from *each* part

Limit answers to the required points.

Max. Marks: 60 Duration: 3 hours

PART A

1. a. Derive the equations of equilibrium in terms of stress components for a 3D stress state. 3

   b. Determine the principal stresses for the state of stress shown below. 4

   $$[\sigma] = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{MPa}$$

   c. For the state of stress shown above, determine the octahedral shear stress at a point. 2

2. a. Derive the strain components at a point in terms of the displacement components for a 3D strain state. 4

   b. The displacement field in a homogeneous isotropic elastic body is given by 5

   $U = A\ \{ (3x^2z+60x)i + (5z^2+10xy)j + (6z^2+2xyz)k \}$, where $A = 1 \times 10^{-6}$ mm. if Youngs modulus is $E = 2 \times 10^5$ MPa, and poisons ratio $\nu = 0.25$, calculate the stress components at the point P(5, 10, -15)mm.

3. a. Derive the traction boundary conditions for a 3D stress state. 4

b. The strain tensor (with the tensorial shearing strains) at a point in a body is given by $\begin{bmatrix} 12 & 3 & 4 \\ 3 & 8 & -4 \\ 4 & -4 & 18 \end{bmatrix} \times 10^{-3}$. Determine the strain tensor with respect to a rotated co-ordinate system, whose direction cosines with respect to the rectangular co-ordinate directions are given by N1= (0.357, -0.206, 0.911), N2= (0.793, 0.583, -0.179) and N3= (-0.494, 0.786, 0.372). Also comment on the results. 5

PART B

4. a. Explain the Generalized Hooke's law. 4

b. Evaluate the coefficients in the Compliance matrix in the strain-stress relations for a homogeneous, linearly elastic material possesses orthotropic elasticity. 5

5. a. A cantilever beam of rectangular cross-section 40 mm wide and 60 mm thick is 800mm in length. The beam carries a load of 500N at the free end. Using the stress function $\varphi = A\left\{xy^3 - \frac{3}{4}xyh^2\right\}$, evaluate the stresses at mid length of the beam, where h is the depth of the beam. 6

b. Explain the case of plane strain problems in theory of elasticity, with example. 3

6. a. Derive the Navier equations for 3D problem of elasticity. 6

b. Show that $\varphi = \{a_1 e^{bx} + a_2 e^{-bx} + a_3 x e^{bx} + a_4 x e^{-bx}\} \cos by$ is a possible stress function for the constants a_1 to a_4 and b. 3

PART C

7. The state of stress for stress distribution in a thick cylinder subjected to internal and external pressures, is expected to be obtained by the stress function $\varphi = A + B log_e r + Cr^2 + Dr^2 log_e r$. Evaluate the constants A, B, C and D and solve for stresses. Assume any missing data suitably. 12

8. Determine the shear stress distribution and warping of the cross-section of a prismatic bar of elliptic cross-section subjected to a torque T. 12

9.	a.	Derive the equations of equilibrium for 2D problems in polar co ordinates	6
	b.	Briefly explain Prandtl's membrane analogy for torsion.	6