No. of Pages: 3 **B**

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER M.TECH. DEGREE EXAMINATION, APRIL/MAY 2018

Branch: Civil Engineering

Stream: Structural Engineering

01CE6104 Finite Element Method

Answer ***any two full*** questions from ***each*** part

Limit answers to the required points.

Max. Marks: 60 Duration: 3 hours

PART A

1. a. The stress components at a point in a body are given by $\sigma_x = 3xy^2z + 2x$, $\sigma_y = 5xyz + 3y$, $\sigma_z = x^2y + y^2z$, $\tau_{xy} = 0$, $\tau_{yz} = \tau_{xz} = 3xy^2z + 2xy$. Check whether these components of stress satisfy the equations of equilibrium or not at the point (1, -1, 2). If not, determine the suitable body force required at this point so that the stress components are under equilibrium. (3)

   b. How far the finite element solution for a problem is satisfied with regard to equilibrium and compatibility? (4)

   c. What does the term 'displacement function' represent in Finite Element Method? (2)

2. a. State and derive the principle of minimum potential energy. (4)

   b. Determine the displacement at the free end of a cantilever beam of span 5 m and subjected to a UDL of 10 kN/m length using Rayleigh -Ritz method and compare the solution with the exact solution. (5)

3. a. A continuous beam, ABCD, has the following details. A, B and C are simple supports with AB = 5 m, BC = 4 m and CD = 1.5 m. Moment of inertia for span AB = Moment of inertia for span BC = 2I, Moment of inertia for CD = I. AB is loaded with a UDL of 5 kN/m, BC is loaded with a concentrated load of 15 kN at 3 m from B and there is a concentrated load of 4 kN at D. Analyse by direct stiffness method. Draw the shear force and bending moment diagrams. (9)

http://www.ktuonline.com

PART B

http://www.ktuonline.com

4. a. A four-node quadrilateral element is having the following Cartesian co-ordinates in cm. Node 1 (1, 1), Node 2 (5, 1), Node 3 (6, 6), Node 4 (1, 4). The element displacement vector in cm is given as $\{u_1 \quad v_1 \quad \ldots\ldots\ldots\ldots u_4 \quad v_4\}^T =$ $\langle 0 \quad 0 \quad 0.2 \quad 0 \quad 0.15 \quad 0.1 \quad 0 \quad 0.05 \rangle$. Determine (i) the x, y co-ordinates of a point 'P' whose location in the parent element is given by $\xi = \eta = 0.5$. (ii). The u, v displacement of point 'P'. (iii). The Jacobian matrix at $\xi = \eta = 0.5$. (5)

b. Under what circumstances a 3D problem can be idealised as a plane stress problem. Give examples. Also write down the respective constitutive relation for a linearly elastic isotropic material. (4)

5. a. In an one dimensional quadratic bar element, the nodal displacements at nodes 1 to 3 are 0.002 cm, 0.004 cm and 0.001 cm respectively, node 3 being the centre of the element. The length of the element is 10 cm. Determine the strain at a point 'P' which is at a distance of L/6 from the left end node (node 1). (4)

a. Develop the shape functions for a quadratic triangular element and plot the shape functions for representative nodes. (5)

6. a. Evaluate the consistent nodal load vector for a beam element of length 3 m and subjected to point loads of 5 kN and 10 kN at distance of 1 m from left end and 1 m from right end respectively. (4)

b. Develop the stiffness matrix for an one-dimensional quadratic bar element of length, 'L' and cross-sectional area 'A'. (5)

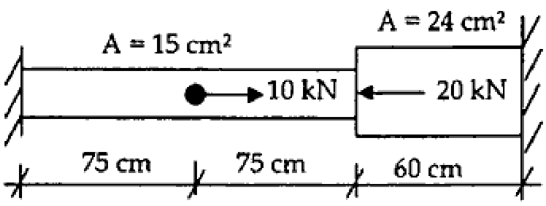
PART C

7. a. Number the nodal points of a rigid-jointed multi-storey frame with 3 bays and 5 storeys so as to attain minimum semi-bandwidth and calculate the same. (2)

b. Explain the penalty approach for imposing the boundary conditions to a problem. (4)

c. Discuss the different types of hexahedral finite elements that can be used for a 3D problem and derive the shape functions for the simplest hexahedral element. (6)

8. a. Determine the displacements at the points of application of loads for the bar shown in figure by finite element method. E = 200 GPa.



(8)

http://www.ktuonline.com

http://www.ktuonline.com

b. Store the following global stiffness matrix by band matrix technique and skyline technique.

$$\frac{29.5x10^6}{600}\begin{bmatrix} 22.68 & 5.76 & -15 & 0 & -7.68 & -5.76 & 0 & 0 \\ & 4.32 & 0 & 0 & -5.76 & -4.32 & 0 & 0 \\ & & 5 & 0 & 0 & 0 & 0 & 0 \\ & & & 20 & 0 & -20 & 0 & 0 \\ & & & & 22.68 & 5.76 & -15 & 0 \\ & \text{symmetric} & & & & 24.32 & 0 & 0 \\ & & & & & & 15 & 0 \\ & & & & & & & 10 \end{bmatrix}$$

(4)

9. a. Solve the following system of equations by LDL^T technique.

$5x_1+3x_2+8x_3 = 67$

$3x_1+9x_2+15x_3 = 120$

$8x_1+15x_2+10x_3 = 134$ (7)

b. Determine the maximum displacement in a cantilever beam of span 'L' and subjected to a UDL of w/ m length by unit load method. (Hint: $\delta = \int_0^L \frac{Mmdx}{EI}$). Use Gauss quadrature technique for the evaluation of integral and comment on the result. (5)