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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

Branch: ***Computer Science and Engineering***

Stream(s):

1. ***Computer Science and Engineering***
2. ***Information Security***

01CS6105: Advanced Data Structures and Algorithms

Answer *any two full* questions from *each* part
Limit answers to the required points.

Max. Marks: 60 Duration: 3 hours

PART A

1. a. Let D be a data structure whose operations have an amortized cost of at most t. 4
   i. Can you give a guaranteed bound on the cost of a sequence of m operations on D? If so, state this bound. Justify your answer.
   ii. Can you give a guaranteed bound on the cost of an individual operation on D? If so, state this bound. Justify your answer.

   b. Suppose we implement a FIFO queue using two stacks called ***InStack*** and ***OutStack*** as follows. An element is inserted into the queue by pushing it into the I***nStack***. An element is extracted from the queue by popping it from the ***OutStack***. If the ***OutStack*** is empty after an extraction operation, then all elements currently in the ***InStack*** are transferred to ***OutStack***, but in the reverse order. Show that a sequence of I insertions and E extractions requires only $O(I+E)$ time, using an appropriate potential function. 5

2. a. Starting from the empty binomial ***max***--heap, perform the following operations 4
   i. Insert 4, 12, 8, 24, 6, 18, and 16, in this order.
   ii. Explain how to delete the ***maximum*** from your resulting ***max***--heap.

   b. Give a sequence of m ***Make-Set***, ***Union***, and ***Find-Set*** operations, n of which are ***Make-Set***, that takes $\Omega(m*log(n))$ time when we use union by rank only. 5

3. a. Consider the ***quick-union*** algorithm for disjoint sets. Explain why if all the finds are done before all the unions, a sequence of n operations is guaranteed to take $O(n)$ time. 3

   b. Find the greatest common divisor **d** of 88 and 16, and find integers **x** and **y** solving the equation $88x + 16y = d$. 2

C. Factor 31861 using the Pollard Rho method with polynomial $f(x) = x^2 + 1$ and initial guess $x_0 = 1$. 4

PART B

4. a. Given a flow network $G = (V, E)$ with source s and sink t, a cut (S,T) is a partition of V into S and $T = V - S$ such that $s \in S, t \in T$. Let (S,T) be a cut of a flow network. Let C be a subset of T such that $t \in C$. Let $D = T - C$. 4

Consider the following statement about the flow network: $f(S, C) + f(T,S) = f(D,S) + f(V,D)$,

where $f(X,Y) = \sum_{x \in X} \sum_{y \in Y} f(x,y)$ and $f(x,y)$ denotes the flow from vertex x to vertex y.

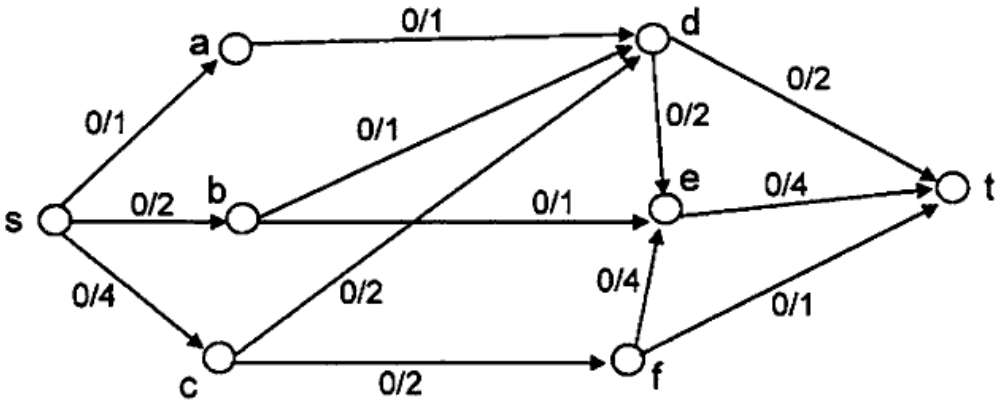
Either prove the statement or provide a counterexample.

b. Working modulo q = 3, how many spurious hits does the Rabin-Karp matcher encounter in the text T = 4126719021586 when looking for the pattern P = 125? 3

c. Define P, NP and NP-complete problems. Give an example for each. 2

5. a. State and prove Maxflow-Mincut theorem. 4

b. Show the result of executing the Ford-Fulkerson algorithm on the flow network below, where node s is the source and node t is the sink. What is the value of the maximum flow from s to t? 3



c. Define *PSPACE* and *NPSPACE*. What can you say about the relationship between *PSPACE* and *NPSPACE*? 2

6. a. Assumes a flow network $G = (V, E)$. Each edge capacity $c(u,v)$ is nonnegative. The source is denoted by s and the sink by t. The value of a flow is denoted by $|f|$. 3

State whether the following statement is TRUE or FALSE. Explain your answer.

$$|f| \leq max(\sum_{v \in V} c(v, t), \sum_{v \in V} c(s, v))$$

b. Compute the prefix function π for the pattern ***abcaabbccab*** when the alphabet is $\Sigma = \{a, b, c\}$. 3

c. Consider the following algorithm to determine whether or not an undirected graph has a clique of size k. First, generate all subsets of the vertices containing exactly k vertices. Next, check whether any of the subgraphs induced by these subsets is complete (i.e. forms a clique). 3
Why is this not a polynomial-time algorithm for the clique problem, thereby implying that $P = NP$?

PART C

7. a. ***QuickSelect*** is the following simple randomized algorithm for finding the k^{th} smallest element in an unsorted set S. 6

QuickSelect(S, k):

1. Pick a pivot element p uniformly at random from S.
2. By comparing p to each element of S, split S into two sets:

   S_1 and S_2, where S_1 contains elements of S less than p and S_2 contains elements of S greater than p.
3. If $|S_1| = k-1$, then output p.

   If $|S_1| > k-1$, then output ***QuickSelect***(S_1, k).

   If $|S_1| < k-1$, then output ***QuickSelect***$(S_2, k-|S_1|-1)$.

Prove that the expected number of comparisons made by ***QuickSelect*** on a set S of size n is at most $4n$.

b. Consider the following problem "Given n line segments, find if any two segments intersect". Give a $O(n*log(n))$ time algorithm that solves the given problem. 6

8. a. Consider tossing m pebbles onto the n nodes of a k-regular undirected graph (a graph is k-regular if every node has degree k). Each pebble lands on a node selected uniformly at random. A pair of pebbles is said to "collide" if they fall on the same node or on two nodes that are neighbors. What is the expected number of pairs of pebbles that collide? About how large must m be before you would expect at least 1 pair of pebbles to collide? 6

b. Consider the following problem "We are given an array of n points in the plane, find out the closest pair of points in the plan". Give a $O(n*log(n))$ time algorithm that solves the given problem. 6

9. a. Define ***RP***, ***co-RP***, ***ZPP*** and ***BPP*** randomized complexity classes. Give at least an example for each. 6

b. Consider the following problem "Given a set of points in the plane, the convex hull of the set is the smallest convex polygon that contains all the points of it". Give a $O(n*log(n))$ time algorithm that solves the given problem. 6