C

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

*Branch: **Computer Science and Engineering***

Stream(s):

1. ***Computer Science and Engineering***
2. ***Information Security***

01CS6105: Advanced Data Structures and Algorithms

Answer *any two full* questions from *each* part

Limit answers to the required points.

Max. Marks: 60 Duration: 3 hours

PART A

1. a. A sequence *n* of operations is performed on a data structure. The ***i-th*** operation costs *i* if *i* is an exact power of 4, and 1 otherwise. 5
   i. Use aggregate analysis to determine the amortized cost per operation.
   ii. Use the accounting method to determine the amortized cost per operation.

   b. Suppose we can insert and delete an element into a hash table in constant time. In order to ensure that our hash table is always big enough, without wasting a lot of memory, we will use the following global rebuilding rules: 4
   - After an insertion, if the table is more than 3/4 full, we allocate a new table twice as big as our current table, insert everything into the new table, and then free the old table.
   - After a deletion, if the table is less than 1/4 full, we allocate a new table half as big as our current table, insert everything into the new table, and then free the old table.

   By defining a potential function, show that for any sequence of insertions and deletions, the amortized time per operation is still constant.

2. a. Starting from the empty binomial *min*--heap, perform the following operations 4
   i. ***Insert(10), Insert(3), Insert(7), Insert(5), RemoveMin, RemoveMin, Insert(6), Insert(20), RemoveMin, Insert(3), Insert(4).***
   ii. Explain how to delete the *minimum* from your resulting *max*--heap.

b. For the following min Fibonacci heap, assume that the ***ChildCut*** field of all nodes is ***TRUE***. 5

```
min
 |
 V
 2-----------------10-------------20
/ | \              | \
4 5 6             11 12
  | \                 |
  7 8                13
    |
    9
```

Figure 1. Min Fibonacci heap

i. Perform a ***DecreaseKey*** operation by changing 8 to 1. Draw the resulting min Fibonacci heap.

ii. Perform a ***Delete 10*** operation on the resulting Fibonacci heap of (I).

3. a. i. Perform the following ***Union-by-Weight*** operations on a universe of 10 elements (0-9, each initially in their own set). Draw the forest of trees that result. ***U(1,5); U(3,7); U(1,4); U(5,7); U(0,8); U(6,9); U(3,9)***. If the sets have equal weight, use the root with the smaller value as the root of the new set. 6

ii. Prove that if ***Union-by-Weight*** is used for all unions, the length of the deepest node is no more than ***log(N)***.

b. Factor 221 using the Pollard Rho method with polynomial $f(x) = x^2 + 1$ and initial guess $x_0 = 2$. 3

PART B

4. a. Given a flow network ***G = (V, E)*** with source ***s*** and sink ***t***, a cut ***(S,T)*** is a partition of ***V*** into ***S*** and ***T = V - S*** such that $s \in S, t \in T$. Let ***(S,T)*** be a cut of a flow network. Let **C** be a subset of ***T*** such that $t \in C$. Let ***D = T - C***. 4

Consider the following statement about the flow network: $f(S, C) + f(T, S) = f(D, S) + f(V, D)$,

where $f(X, Y) = \sum_{x \in X} \sum_{y \in Y} f(x, y)$ and $f(x, y)$ denotes the flow from vertex ***x*** to vertex ***y***.

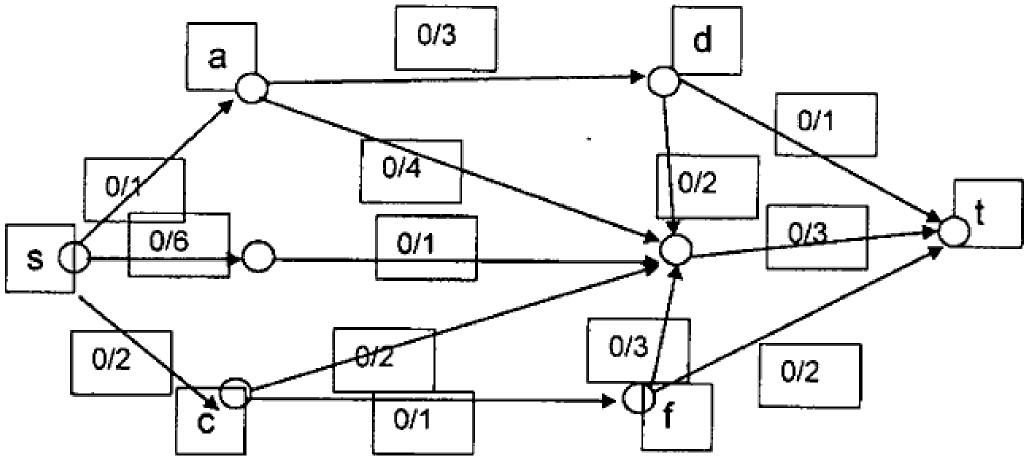
Either prove the statement or provide a counterexample.

b. Working modulo ***q = 13***, how many spurious hits does the Rabin-Karp matcher encounter in the text ***T = 2359023141526739921*** when looking for the pattern ***P = 12531415***? 3

c. Define P, NP and NP-complete problems. Give an example for each. 2

5. a. State and prove Maxflow-mincut theorem. 4

b. Show the result of executing the Ford-Fulkerson algorithm on the flow network below, where node *s* is the source and node *t* is the sink. What is the value of the maximum flow from *s* to *t*? 3



c. Define ***PSPACE*** and ***NPSPACE***. What can you say about the relationship between *PSPACE* and *NPSPACE*? 2

6. a. Assumes a flow network ***G = (V, E)***. Each edge capacity ***c(u,v)*** is nonnegative. The source is denoted by *s* and the sink by *t*. The value of a flow is denoted by $|f|$. 3

State whether the following statement is TRUE or FALSE. Explain your answer.

$$|f| \leq \max\left(\sum_{v \in V} c(v,t), \sum_{v \in V} c(s,v)\right)$$

b. For the pattern ***P=abababAbca***, compute the prefix function. For ***T= cabacababababcababababcac***, run the ***KMP*** algorithm for searching **P** within *T*. Show the traces of your work, not just the final results. 3

c. Consider the following algorithm to determine whether or not an undirected graph has a clique of size *k*. First, generate all subsets of the vertices containing exactly *k* vertices. Next, check whether any of the subgraphs induced by these subsets is complete (i.e. forms a clique). 3
Why is this not a polynomial-time algorithm for the clique problem, thereby implying that *P = NP*?

PART C

7. a. Suppose that *A* is a polynomial-time randomized algorithm for Problem ***X***, whose "yes" answers are always correct, and that on any member of ***X***, ***A*** answers "yes" with probability at least ***1/n²***. Show that, there is a polynomial-time randomized decision procedure ***B*** for ***X*** that is correct with probability at least 3/4 on any input. 6

b. Consider the following problem "Given *n* line segments, find if any two segments intersect". Give a *O(n*log(n))* time algorithm that solves the given problem. 6

8. a. Consider tossing *m* pebbles onto the *n* nodes of a *k*-regular undirected graph (a graph is *k*-regular if every node has degree *k*). Each pebble lands on a node selected uniformly at random. A pair of pebbles is said to "collide" if they fall on the same node or on two nodes that are neighbors. What is the expected number of pairs of pebbles that collide? About how large must *m* be before you would expect at least 1 pair of pebbles to collide? 6

b. Consider the following problem "We are given an array of *n* points in the plane, find out the closest pair of points in the plan". Give a *O(n*log(n))* time algorithm that solves the given problem. 6

9. a. Define *RP, co-RP, ZPP* and *BPP* randomized complexity classes. Give at least one example for each. 6

b. Consider the following problem "Given a set of points in the plane, the convex hull of the set is the smallest convex polygon that contains all the points of it". Give a *O(n*log(n))* time algorithm that solves the given problem. 6