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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

Branches: *Computer Science & Engineering & Information Technology*

Streams :

1. *Computer Science & Engineering*
2. *Information Security*
3. *Network Engineering*

01 CS 6101: Mathematical Foundations of Computing Systems

Answer any two full questions from each part

Limit answers to the required points.

Max. Marks: 60

Duration: 3 hours

PART A

1. a. For $a \in \mathbb{Z}$, if $a^2 - 2a + 7$ is even, then a is odd. Prove the statement by contradiction and contrapositive proof techniques. (5)
b. Using complete induction show that if n is an integer greater than 1, then n can be written as the product of primes. (4)
2. a. Prove that $\sqrt{3}$ is irrational. (5)
b. Using mathematical induction prove that $1^3 + 2^3 + \dots + n^3 = (1 + 2 + \dots + n)^2$ for $n \in \mathbb{Z}^+$ (4)
3. a. Write short notes on linear time temporal logic. (4)
b. Using generating function, solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$, with initial condition $a_0 = a_1 = 3$. (5)

PART B

4. a. Every sequence of $(n^2 + 1)$ distinct real numbers contain a subsequence of length $(n + 1)$ that is either strictly increasing or strictly decreasing. Prove using pigeon hole principle. (5)
b. From a box of 3 black and 4 white marbles,
(i) If two marbles are drawn successively, find the probability that both are black if the first marble is not replaced before the second drawing.
(ii) If two marbles are drawn successively, find the probability that both are black if the first marble is replaced before the second drawing.
(iii) If four marbles are drawn at random, find the probability that there are equal number of marbles of both colors.
(iv) If three marbles are drawn successively without replacement, find the (4)

probability that none of them are white.

5. a. How many ways are there to choose 6 items from 10 distinct items when
 - (i) the items in the choices are ordered and repetition is not allowed?
 - (ii) the items in the choices are ordered and repetition is allowed? (4)
 - (iii) the items in the choices are unordered and repetition is not allowed?
 - (iv) the items in the choices are unordered and repetition is allowed?
- b. Admission to a foreign university is determined by an entrance examination, with its scores normally distributed with a mean of 500 and a standard deviation of 100. A student will be admitted to the university if he scores better than at least 70% of the students who wrote the entrance. (5)
 - (i) Alice scored 585 marks. Will he be admitted to the university?
 - (ii) Bob says he will not get admission in the university. What will be his marks? <http://www.ktuonline.com>
6. a. Show that in a group of 10 people (where any two people are either friends or enemies), there are either three mutual friends or four mutual enemies and there are either three mutual enemies or four mutual friends. (5)
- b. (i) Explain continuous and discrete random variables. (4)
- (ii) Write the properties of a standard normal curve.

PART C

7. a. Define the following with an example: (6)
 - (i) n- dimensional hypercube
 - (ii) planar graph
 - (iii) decision tree
 - (iv) bipartite graph
- b. If G is a group whose table is given below. Show that $H=\{1,c,d\}$ and $K=\{1,b\}$ are both subgroups of G. Find the left coset aH and the right coset Ka .

	1	a	b	c	d	e
1	1	a	b	c	d	e
a	a	1	c	b	e	d
b	b	d	1	e	a	c
c	c	e	a	d	1	b
d	d	b	e	1	c	a
e	e	c	d	a	b	1

8. a. State and prove 5-color theorem. (6)

- b. Define the following
- (i) Generating set of a group. (3)
 - (ii) Cyclic group
 - (iii) homomorphism
- c. Prove that $Z_n = \{0, 1, 2, \dots, n-1\}$ is an abelian group under addition modulo n . (3)
9. a. (i) An undirected graph has an even number of odd degree vertices. Prove (3)
- b. G_1 is K_m (complete graph), G_2 is $K_{m,n}$ (complete bipartite graph), G_3 is C_n (cycle) and G_4 is a graph with k isolated vertices (3)
- (i) Find the chromatic number of G_3 .
 - (ii) Find the chromatic polynomial of K_m and $K_{m,n}$.
 - (iii) Find the chromatic number and chromatic polynomial of G , if G is a graph with G_1 and G_4 as its components.
- c. Explain reciprocity and quadratic residues. (6)