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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017
Branches: Computer Science & Engineering & Information Technology

Streams :

1. *Computer Science & Engineering*
2. *Information Security*
3. *Network Engineering*

01 CS 6101: Mathematical Foundations of Computing Systems

Answer any two full questions from each part

Limit answers to the required points.

Max. Marks: 60

Duration: 3 hours

PART A

1. a. If n is an even integer and $3n+2$ is even, then n is even. Prove using contradiction and contraposition proof techniques. (5)
b. Every positive integer n can be written as the sum of distinct powers of 2. Prove using strong induction. (4)
2. a. Solve the recurrence relation $a_n - 4a_{n-1} = 6 \cdot 4^n$, $a_0 = 1$ using generating functions. (5)
b. Explain branching time logic. (4)
3. a. A simple polygon with t sides, where $t \in \mathbb{Z}$ and $t \geq 3$, can be triangulated into $(t-2)$ triangles. Prove using complete induction. (5)
b. If $n \in \mathbb{Z}^+$, then n is even if and only if $7n+4$ is even. Prove (4)

PART B

4. a. Show that in a sequence of m integers, there exists one or more consecutive terms with a sum divisible by m . (5)
b. (i) State Bayes Theorem. (2)
(ii) Given $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(B|A) = 1$, find $P(A|B)$.
Also $P(A_1) = \frac{3}{4}$, $P(B_1) = \frac{1}{4}$ and $P(A_1|B_1) = \frac{1}{2}$. Find $P(B_1|A_1)$. (2)
5. a. Using Cantor's diagonalization argument, prove that the set of real numbers is uncountable. (4)
b. (i) A fair coin is tossed 4 times. Find the probability that they are all heads if the first two tosses result in heads. (5)
(ii) Let A and B be two mutually exclusive events of an experiment. If $P(A^c) = 0.65$, $P(A \cup B) = 0.65$, find $P(B)$.

(iii) The probability that a shooter hits the target is $1/4$ and he shoots 60 times. Find the expected number of times he hits the target and the standard deviation.

(iv) In the standard normal curve, the area between 0 and z is 0.4484. Find z .

(v) A fair coin is tossed 2 times. How many would you expect it falls head?

6. a. (i) In how many different ways can the letters of the word DETAIL be arranged such that the vowels must occupy only the *odd* positions? (5)
- (ii) How many factors of $(2^3 \times 3^6 \times 5^2)$ are perfect squares?
- (iii) How many 8 bit strings contain exactly four ones?
- (iv) Determine the number of integers between 1 and 10000 that are not divisible by 6, 7 or 8.
- b. The number of complaints registered at a service station per minute is a Poisson variable with mean=3. (4)
- (i) Find the probability that a given period of 1 minute is complaint free.
- (ii) Assume that the number of complaints received in two different minutes are independent. Find the probability that at least two complaints will be registered in a given 2- minute period.

PART C

7. a. Define chromatic number and chromatic polynomial. (2)
- b. G_1 is K_m (complete graph), G_2 is $K_{m,n}$ (complete bipartite graph), G_3 is C_{2k+1} (cycle), G_4 is C_{2k} (cycle) (4)
- (i) Find the chromatic number of G_1, G_2, G_3 and G_4 .
- (ii) Find the chromatic polynomial of G_1, G_2, G_3 and G_4 .
- (iii) G is a graph with G_1 and G_3 as components. Find the chromatic number and chromatic polynomial of G .
- c. Write short notes on Elliptic curve arithmetic. (6)
8. a. Explain Warshall's algorithm with its complexity. (6)
- b. Define left coset and right coset. Give an example for each. (2)
- c. Let $(G,*)$ be a group, $(S,*)$ be a subgroup, S_1 and S_2 are two left cosets of S . Then either S_1 and S_2 are disjoint or they are identical. Prove (4)
9. a. A connected planar graph with n vertices and e edges has $e-n+2$ regions. Prove (6)
- b. State and prove Lagrange's theorem. (3)
- c. Define ring. Give an example each for commutative ring and ring with identity. (3)