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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017
Electronics and Communication Engineering
1.Signal Processing
2. Telecommunication Engineering
01 EC6301 Applied Linear Algebra

Max. Marks : 60 **Duration: 3 Hours**

Answer any two questions from each part.
Limit answers to the required points.

PART A

1 a) Prove that the identity element of a group G is unique and the inverse of each element of G is unique. (4 marks)

b) Solve by Gauss Elimination method

$$2x + y + 2z + w = 6, \quad x - y + z + 2w = 6,$$
$$4x + 3y + 3z - 3w = -1, \quad 2x + 2y - z + w = 10$$ (5 marks)

2 a)Define subspace of a vector space. Prove that the intersection of two subspaces of a vector space is a subspace. (4 marks)

b) Find a spanning set for the Null space of, where

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$ (5 marks)

3 a) Prove that a subset S of a vector space V is a basis of V if and only if S is a minimal generating set. (4 marks)

b) Find theleast square solution of the system $AX = B$ where $A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix}$ and

$B = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}$ Also compute the least square error. (5 marks)

PART B

4 a) Define kernel of a linear transformation. Show that kernel of a linear transformation $T: V \rightarrow W$ is a subspace of V. (4 marks)

b) Show that the function T from $\boldsymbol{R}^3$ into $\boldsymbol{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1)$ is a linear transformation. Find the matrix of T relative to the standard bases of $\boldsymbol{R}^3$ and $\boldsymbol{R}^2$. (5 marks)

5 a) Find an orthonormal basis for the subspace spanned by the vectors $(1,2,1)$, $(1,0,1)$ *and* $(3,1,0)$ in $\boldsymbol{R}^3$ (4 marks)

b) State and prove rank nullity theorem. (5 marks)

6 a) Let $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ be bases for $\boldsymbol{R}^2$ where $b_1 = (7,5)$, $b_2 = (-3,-1)$, $c_1 = (1,-5)$ *and* $c_2 = (-2,2)$. Find the transition matrix from B to C and from C to B. (4 marks)

b) State and prove Cauchy - Schwarz inequality. (5 marks)

PART C

7 a) Find the eigen values of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ Also find a basis for the eigen space of each eigen value. (6 marks)

b) Find the singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$ (6 marks)

8 a) If possible, diagonalize $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$ (6 marks)

b) Define a Hermitian matrix. Prove that the eigen values of a Hermitian matrix are real. (6 marks)

9 a) Construct a spectral decomposition of $A = \begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}$ (6 marks)

b) Examine the definiteness of the quadratic form $Q(x) = 3x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 + 4x_2x_3$ (6 marks)