No: of pages: 2 **A**

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

Electronics and Communication Engineering

1. Signal Processing

2. Telecommunication Engineering

01 EC 6301 Applied Linear Algebra

Max. Marks : 60 **Duration: 3 Hours**

Answer any two questions from each part.

Limit answers to the required points.

PART A

1 a) Determine whether the set of all integers $\boldsymbol{Z}$ is a group under multiplication. Justify your answer.(4 marks)

b) Show that the equations $ax = b$ and $ya = b$ have unique solutions in a group G for all $a, b \in G$ (5 marks)

2 a) Check whether the vectors $v_1 = (2.-1,3)$, $v_2 = (1,0,-1)$ and $v_3 = (2,4,1)$ in $\boldsymbol{R}^3$ are linearly independent.(4 marks)

b) Solve by Gauss Elimination method $x + 2y + z = 3,\ 2x + 3y + 3z = 10,\quad 3x - y + 2z = 13$.(5 marks)

3 a) If V is a vector space of dimension n, prove that any set of n linearly independent elements of V is a basis of V (4 marks)

b) Find the least square solution of the system

$$AX = B, where\ A = \begin{bmatrix} 1 & -3 & -3 \\ 1 & 5 & 1 \\ 1 & 7 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ -3 \\ -5 \end{bmatrix}$$ (5 marks)

PART B

4 a) Define range of a linear transformation. Show that the range of a linear

transformation$T: V \rightarrow W$ is a subspace of W (4 marks)

b) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ is the matrix in the standard ordered basis of a linear operator$T$ on$\boldsymbol{R}^3$, find a basis for the null space of T(5 marks)

5 a) Define an inner product space with an example.(4 marks)

b) Let $B = \{b_1, b_2\}$ *and* $C = \{c_1, c_2\}$ be bases for a vector space V and let $b_1 = -c_1 + 4c_2, b_2 = 5c_1 - 3c_2$. Find the change of coordinates matrixfrom$B$ to$C$ and the C coordinate vector of $x = 5b_1 + 3b_2$.
(5 marks)

6 a) Show that an orthonormal set of vectors is linearly independent (4 marks)

b) Find an orthonormal basis for $\boldsymbol{R}^3$ starting with the vector (1,1, −1)(5 marks)

PART C

7 a) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$(7 marks)

b) If λ is an eigen value of an orthogonal matrix A prove that $\frac{1}{\lambda}$ is also an eigen value of A(5 marks)

8 a) Orthogonallydiagonalize the matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ (7 marks)

b) State spectral theorem.(5 marks)

9 a) Find the singular value decomposition of $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$(7 marks)

b) Find a basis for the eigen space corresponding to the eigen value = 3 of

$A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$(5 marks)