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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2015

Electronics And Communication Engineering

(Signal Processing)

01EC6303 Random Process and Applications

Max Marks: 60 Duration: 3 Hours

Answer two questions from each part

PART A

1. a. Consider a discrete random variable

$$F_X(x) = \sum_{k=0}^{x} \binom{n}{k} p^k(1-p)^{n-k}$$

Plot the cdf for p=0.6and n=4.
Find $P[1.5 < x < 3]$, $P[1.2 < x \leq 1.8]$ (3)

b. Five missiles are fired against an aircraft carrier in the ocean. It takes at least two direct hits to sink the carrier. All the five missiles are on the correct trajectory but must get through the "point defence" guns of the carrier. The point defence guns can destroy a missile with probability 0.9. What is the probability that the carrier will still be afloat after the encounter? (3)

c. The distribution function of a random variable X is given by
$F(x) = 1-(1+x)e^{-x}$ for $x\geq 0$. Find the density function, mean and variance of x. (3)

2. a. State and Prove Baye's theorem (2)

b. $f_{xy}(x,y) = c(x+y)$, $0\leq x\leq 1, 0\leq y\leq 1$.
Find c and $P(X+Y\leq 1)$. Are X and Y independent? (3)

c. Let θ be a prescribed angle and X and Y be the random variables. Consider the rotational transformation
$v = X\cos\theta + Y\sin\theta$
$w = X\sin\theta - Y\cos\theta$
If $f_{XY}(x,y) = 1/(2\pi\sigma^2) \exp[-(x^2+y^2)/2\sigma^2]$, find $f_{VW}(v,w)$ (4)

3. a. $y = \sin x$ and $f_X(x) = \begin{cases} \frac{1}{2\pi} & -\pi \leq x \leq \pi \\ 0 & otherwise \end{cases}$

Find fy(y). (3)

b. X and Y are independent random variables with $X = Y \sim U(0,1)$. Find and plot the pdf of $Z = \max(X, Y)$. (3)

c. Prove that if X and Y are zero mean Gaussian random distributions $\sqrt{x^2 + y^2}$ will have Rayleigh distribution (3)

PART B

4. a. Find the characteristic function of $N(\mu,\sigma 2)$ and hence find mean and variance using it. (3)

b. Find the correlation coefficient of (3)

$$f_{XY}(x,y) = \begin{cases} 24xy\,, & x > 0, y > 0, x + y \leq 1 \\ 0 & , otherwise \end{cases}$$

c. The two independent random variables X and Y have the variances 36 and 16, find the correlation coefficient between X+Y and X-Y (2)

5. a. A random vector $X = [X1,X2,X3]^T$ has a covariance matrix $K = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

Design a non –trivial transform that will generate from X a new random vector Y, whose components are uncorrelated. (3)

b. The transition probability matrix of a markov chain with 3 states 0,1,2 is

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

Initial state distribution of the chain is $P[X_0 = i] = 1/3$ for $i = 0,1,2$.Find

i. $P(X_2 = 2)$ ii. $P(X_1=1/X_2=2)$

iii. $P(X_3=1, X_2=2, X_1=1, X_0=2)$ iv. $P(X_2=2,X_1=1/X_0=2)$ (3)

c. Find the autocorrelation function of the Poisson Process and hence find the covariance. (2)

6. a. If $X_1(t)$ and $X_2(t)$ represent two independent Poisson Process with parameter $\lambda_1 t$ and $\lambda_2 t$, then show that $X_1(t) + X_2(t)$ is also Poisson Process with parameter $(\lambda_1 + \lambda_2)t$ while $X_1(t) - X_2(t)$ is not a Poisson Process. (3)

b. X(t) is a Gaussian process with $\mu(t) = 10$ and $c(t1,t2) = 16e^{-|t1-t2|}$ (3)
find probability that $X(10) \leq 8$ and also find the probability that $|X(10) - X(6)| \leq 4$

c. What is random walk and how it can be approximated to a wiener process. Find mean and variance of random walk. (2)

PART C

7. a. State and Prove Chernoff Bound and Schwarz inequality (5)

b. A post office handles 10,000 letters per day with a variance of 2000 letters. What

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can be said about the probability that this post office handles between 8000 and

12000 letters tomorrow. What about the probability that more than 15000 letters come in. (3)

c. A WSS process X(n) is to be generated with $RXX(0) = \sigma^2$ and $RXX(1) = \rho\sigma^2$ by passing white noise process with unit variance through a system described by a stochastic differential equation X(n) = aX(n-1) +bW(n). Find a and b. (4)

8. a. $\hat{X}(t) = \sum_{n=1}^{\infty} X_n(t)\phi_n(t)$ converges to X(t) in mean square sense where the set of orthonormal function $\{\phi n(t)\}$ are the solution of the integral equation[-T/2, +T/2] .

$$\int_{-T/2}^{+T/2} Rxx(t1,t2)\phi n(t2)dt2 = \lambda n\ \phi n(t1)$$

(5)

b. Also prove the coefficient of the random variable X_n are statistically orthogonal (2)

c. Derive the sufficient condition for the random process X(t) to be ergodic in mean (5)

9. a. If the WSS process X(t) is mean square periodic, then X(t) = X(t+T) in the mean square sense. (3)

b. Prove that

$$X(t) = \sum_{n=-\infty}^{n=\infty} A_n \exp(jnw_0 t)$$

in the mean square sense (4)

c. What is convergence in distribution and convergence in probability? Consider a random sequence $\{X_n\}$, n=1,2,... where $P[X_n=0] = 1 - (1/n)$ and $P[X_n = 1] = 1/n$

Check whether the random sequence X_n converges to zero in

1) Distribution 2) probability 3) mean square (5)