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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

Electronics and Communication Engineering

1. **Signal Processing**
2. **Microwave and TV Engineering**
3. **Telecommunication Engineering**

01EC6303 Random Processes and Applications

Answer *any two full* questions from *each* part

Max. Marks: 60 Duration: 3 hours

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PART A

1. a. State and prove the total probability theorem. 4

   b. Let X, Y be independent random variables with probability density functions $e^{-x}u(x)$ and $e^{-y}u(y)$ respectively. Find the joint probability density function of $Z = \frac{X}{X+Y}$ and $W = X + Y$. 5

2. a. Obtain mean and moment generating function of a Geometric random variable with parameter p. 4

   b. The joint probability density function of two random variables X and Y is given by 5

$$f_{X,Y}(x,y) = \begin{cases} k(x+y), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & otherwise. \end{cases}$$

   Find (i) the value of k

   (ii) the conditional probability density function $f_{X|Y}(x|y)$

   (iii) P(X > 0.2 | y=0.7)

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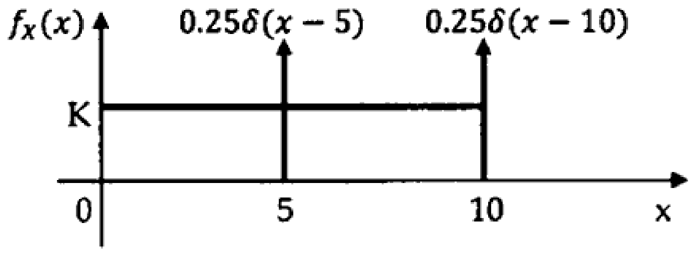
3. a. The probability density function of a random variable X is shown in the figure below. 3



Find (i) the constant K

(ii) Compute $P(X \leq 5), P(5 \leq X < 10)$

(iii) Draw the cumulative distribution function

b. (i) If X is a uniform random variable in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then find the probability density function of Y = tan(X). 6

(ii) If X and Y are independent and identically distributed uniform random variables in the interval $[0, 1]$, then find the probability density function of Z = max(X, Y).

PART B

4. a. Compute the joint characteristic function of two random variables X and Y described by the joint probability density function 3

$$f_X(x, y) = \frac{1}{2\pi} exp\left\{-\frac{x^2 + y^2}{2}\right\}$$

b. Determine the expectation vector and covariance matrix of a two dimensional random vector $[X_1\, X_2]^T$ described by its joint probability density function 6

$$f_{X_1 X_2}(x_1, x_2) = \begin{cases} 2, & 0 \leq x_1 \leq x_2 \leq 1 \\ 0 & otherwise. \end{cases}$$

5. a. Define the following 9

(i) Poisson counting process

(ii) Wiener process

(iii) Birth death Markov chain

6. a. If X and Y are two continuous random variables, then show that $E[E[Y|X]] = E[Y]$. 4

b. A white Gaussian noise process W(t) with two-sided power spectral density $\frac{N_0}{2}$ is given as input to an ideal low pass filter with cutoff frequency B Hz and unity gain. Find the power spectral density and autocorrelation of the output random process. 5

PART C

7. a. State and prove Chebyshev inequality. 6

   b. A normal random variable X has mean value of 5.5 and variance 1. Find an estimate of $P(X \geq 11)$ using Chernoff bound. 6

8. a. Define the following for a sequence of random variables 4

   (i) Almost sure convergence

   (ii) Convergence in probability

   (iii) Mean-square convergence

   (iv) Convergence in distribution

   b. State and prove Central Limit Theorem for independent and identically distributed random variables. 8

9. a. The transition probability matrix P of a two-state Markov chain is given by 3

$$P = \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}.$$

   Find the steady state distribution of the chain.

   b. Derive the Karhunen-Loeve (KL) expansion of a Wiener random process. 9