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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2018

Electronics and Communication Engineering

Stream(s): **1. Signal Processing**

2. Microwave & TV Engineering

3.Telecommunication Engineering

01EC6303 Random Processes and Applications

Answer *any two full* questions from *each* part

Max. Marks: 60 Duration: 3 hours

PART A

1. a. State and Prove Bayes' theorem in probability. 3

   b. If P[A]=0.5, P[B]=0.3 and $P[A \cap B] = 0.15$, then find $P[A/B^c]$. 3

   c. If X is a binomially distributed random variable with E[X]=2 and var[X]=4/3 , find P[X=5]. 3

2. a. The joint pdf of two random variables X and Y is given by 5

$$f_{XY}(x,y) = c \qquad 1 < x < 3, 2 < y < 4$$
$$= 0 \qquad otherwise$$

Find i) The value of c

ii) P[1<X<2, 3<Y<4]

iii) P[X+Y≤5]

   b. Obtain the mean and the moment generating function of a Poisson random variable with parameter λ. 4

3. a. If X is a standard normal random variable, then find the pdf of $Y = e^X$. 5

b. If X and Y are independent and identically distributed uniform random variables in the interval [0, 1], then find the pdf of $Z = \max(X, Y)$. 4

PART B

4. a. Determine the expectation vector and covariance matrix of a two dimensional random vector $[X_1\ X_2]^T$ described by the joint probability density function 5

$$f_{X_1X_2}(x_1, x_2) = \begin{cases} 2 & 0 \le x_1 \le x_2 \le 1 \\ 0 & otherwise. \end{cases}$$

b. The joint probability density function of two random variables X and Y is given by 4

$$f_{XY}(x, y) = \begin{cases} 8xy, & 0 \le x \le 1, 0 \le y \le x \\ 0, & otherwise. \end{cases}$$

Find the covariance of X and Y.

5. a. Explain a Poisson counting process. 6

b. Prove that the interarrival times of a Poisson counting process follows an exponential distribution. 3

6. a. Find the mean and variance of a continuous random variable with moment generating function 4

$$M_X(s) = \frac{3}{3-s}$$

b. A WSS random process X(t) with autocorrelation function $R_{XX}(\sigma)=2\delta(\sigma)$ is the input to a continuous time LTI system with impulse response $h(t)=e^{-t}u(t)$. Find the power spectral density and the autocorrelation of the output random process Y(t). 5

PART C

7. a. State and prove Schwarz inequality for random variables. 4

b. A random variable X has **E[X]=8, var[X]=9** and an unknown probability distribution. Find 4

i) $P[|X-8| \ge 6]$

ii) $P[-4 < X < 20]$

c. Define the following for a sequence of random variables 4

(i) Almost sure convergence

(ii) Convergence in probability

(iii) Convergence in Mean-square sense

(iv) Convergence in distribution

8. a. The Transition Probability Matrix of a three state Markov chain is 6

$$P= \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{pmatrix}$$

Find the steady state distribution of the chain.

b. State and Prove the Weak law of large numbers. 6

9. a. Derive KL expansion of a standard Wiener random process. 8

b. A real valued band pass random process is represented as 4

$U(t) = X(t)cos\omega_0(t) - Y(t)sin\omega_0(t)$.

Explain a method to obtain low pass random processes X(t) and Y(t).