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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, JULY 2018

Electronics and Communication Engineering

(SIGNAL PROCESSING)

01EC6303 Random Processes and Applications

Time: 3 hrs Maximum marks: 60

Answer two questions from each part

Part A

1. a) State and explain Bayes' theorem in probability. (3)

   b) Let X and Y are independent uniform random variables in the interval [0,1]. Find the joint probability density function $f_{ZW}(z,w)$ of Z and W, where Z and W are given as Z =X/Y and W=XY (6)

2. a) Find the mean and variance of a continuous random variable with moment generating function $\emptyset_X(s) = \frac{3}{3-s}$ (4)

   b) The joint probability density function of two random variables X and Y is given by $f_{XY}(x,y) = \begin{cases} e^{-(x+y)}, x \geq 0, y \geq 0 \\ 0 \quad , otherwise \end{cases}$

   Find i) P[X<1] ii) P[(X+Y)<1] (5)

3. a) A continuous random variable X is described by the cumulative distribution function

$$F_X(x) = \begin{cases} 0 & , x < 0 \\ kx^2 & , 0 \leq x \leq 10 \\ 100k & , x > 10 \end{cases}$$

   Find i) The value of k ii) P[X>7] (3)

   b) If X is a uniform random variable in the interval [0,2π] ,then find the probability density function of Y=cos(X) (6)

Part-B

4. A two dimensional random vector $[X \;\; Y]^T$ is described by the probability density function

$$f_{XY}(x,y) = \begin{cases} 2 & ,0 < x < y < 1 \\ 0 & ,otherwise \end{cases}$$

Find the correlation coefficient between X and Y. (9)

5. Define the following
   i) Wiener process
   ii) Poisson process
   iii) Birth –Death Markov chain (9)

6. a) Consider the random process $X(t)=A\cos(\omega_0 t+\theta)$, where A and ω_0 are constants, θ is a uniformly distributed random variable in the interval $(-\pi,\pi)$. Find the mean and auto correlation of the random process X(t). (5)

   b) If X and Y are two discrete random variables, then show that
   $E[E(X/Y)] = E(X)$ (4).

Part-C

7. a) State and prove Markov Inequality (6)

   b) A continuous random variable X has mean $\bar{X}=12$ and variance $\sigma_x^2 = 9$ and has an unknown probability distribution. Find the lower bound for the probability of the event $P(6<x<18)$ using Chebyshev Inequality. (6)

8. a) Show that the convergence of a random sequence with probability-1 implies the convergence in probability. (6)

   b) State and prove weak law of large numbers. (6)

9. a) The transition probability matrix (tpm) of a Markov chain having three states 1,2 and 3 is

$$P= \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{pmatrix}$$

The initial probability distribution is P(0)=[0.5 0.3 0.2]. Find the following

i) $P[X_2 = 2]$

ii) $P[X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3]$ (8)

b) Derive the Karhunen-Loeve (KL) expansion of white noise process (4)
