

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY  
SECOND SEMESTER M.TECH DEGREE EXAMINATION, MAY 2016  
Electrical and Electronics Engineering  
Control Systems, Guidance and Navigational Control

01EE6104: NONLINEAR CONTROL SYSTEMS

Time: 3 hrs

Max. Marks: 60

*Answer any two full questions from each part*

**PART A**

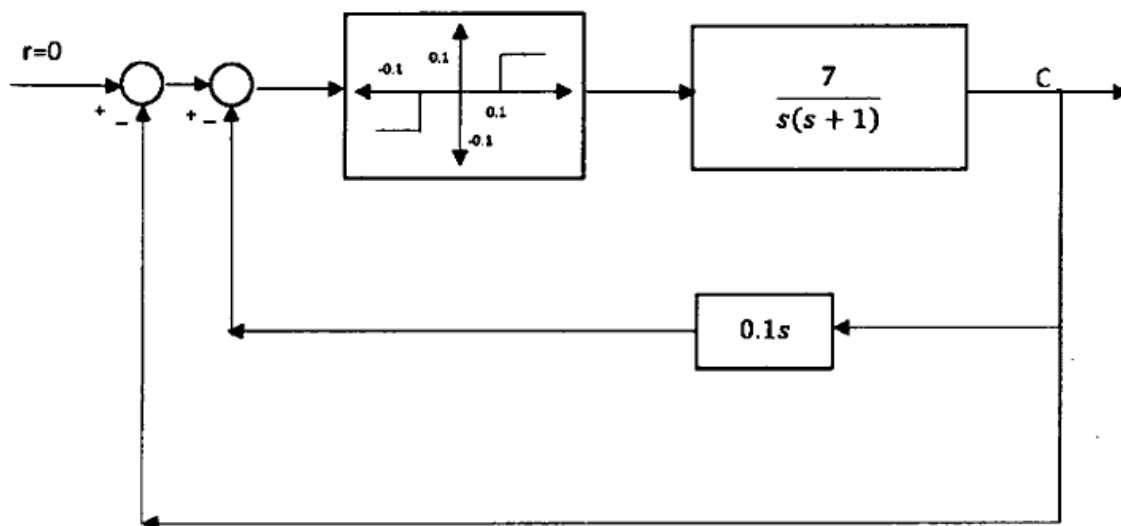
1. (a) Find all equilibrium points of the system (3)

$$\dot{x}_1 = -x_1 + ax_2 - bx_1x_2 + x_2^2$$

$$\dot{x}_2 = -(a+b)x_1 + bx_1^2 - x_1x_2$$

where  $a > 0$  and  $b \neq 0$

- (b) Determine the type of each isolated equilibrium point for all values of  $a > 0$  and  $b \neq 0$ . (3)
- (c) Construct the phase portrait and discuss the qualitative behavior of the system when  $a = b = 1$ . (3)
2. Construct an approximate phase trajectory for the following nonlinear system. Choose appropriate initial conditions. (9)



3. (a) State and explain the theorems (local and global) on the uniqueness and existence of solutions. (3)
- (b) State and prove the theorem on continuity of solutions in terms of initial states and parameters. (6)

**PART B**

4. Explain
- (a) Chetaev's Instability theorem. (3)
  - (b) Aizermann's and Kalman's conjecture. (3)
  - (c) Kalman Yakubovich Popov Lemma. (3)
5. Define stability in the sense of Lyapunov. State and prove Lyapunov's theorem on stability. (9)
6. (a) Find the sector  $[\alpha, \beta]$  for which the system with feedback nonlinearity is absolutely stable using Popov criterion. The forward transfer function of the system is (6)
- $$G(s) = \frac{s}{(s^2 - s + 1)}$$
- (b) State the conditions to be satisfied by a transfer function matrix to be strictly positive real. (3)

**PART C**

7. (a) Explain in detail the concept gain scheduling and the steps involved in the development of a gain scheduled tracking controller for nonlinear systems. (4)
- (b) Consider the system (8)
- $$\begin{aligned}\dot{x}_1 &= x_1 x_2 \\ \dot{x}_2 &= x_1 + u\end{aligned}$$
- Design a feedback control and a change of variable that linearize the system and place the poles at  $-2 \pm j1$
8. (a) Explain diffeomorphism. (3)
- (b) Consider the system (9)
- $$\begin{aligned}\dot{x}_1 &= e^{x_2} - 1 \\ \dot{x}_2 &= a x_1^2 + u\end{aligned}$$
- Is this system feedback linearizable? If yes, find a feedback control law that linearize the state equation.
9. (a) Explain the design procedure of a backstepping controller for a nonlinear system. (5)
- (b) Given the system (7)
- $$\begin{aligned}\dot{x}_1 &= x_2 + \theta x_1^2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= u\end{aligned}$$
- where  $\theta \in [-1, 1]$ . Design a backstepping controller.