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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
Second Semester M.Tech Degree Examination, May 2016
Branch: Electrical and Electronics Engineering
Streams: Control Systems, Guidance and Navigational Control

01EE6102: OPTIMAL CONTROL THEORY

Time: 3 hrs Max. Marks: 60

Instruction: Answer any two full questions from each part

PART A

1. (a) How will you formulate an optimal control problem? How the performance measure is selected in various physical problems? (4)

   (b) The pitch angle $\theta(t)$ of a manned spacecraft is to be controlled by a gas expulsion system. The differential equation that describes the motion is (5)

$$I\frac{d^2\theta(t)}{dt^2} = \lambda(t)$$

where I is the moment of inertia and $\lambda(t)$ is the torque produced by gas jets. Selecting $x_1(t) = \theta(t)$ and $x_2(t) = \dot{\theta}(t)$ as state variables and $u(t) = \lambda(t)/I$ as the control gives the state equations as

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = u(t)$$

The primary objective of the control system is to maintain the angular position near zero. This is to be accomplished with small acceleration (control input). Formulate the optimal control problem.

2. (a) Derive Weierstrass- Erdmann corner conditions for piecewise smooth extremals. (6)

   (b) Find the necessary conditions that must be satisfied by the curve of smallest length which lies on the sphere $w_1^2(t) + w_2^2(t) + t^2 = R^2$ for $t \in [t_0, t_f]$ and joins the specified points w_0, t_0 and w_f and t_f (3)

3. (a) Find the extremal of (4)

$$J(x) = \int_0^1 [x^2(t) + \dot{x}^2(t)]dt$$

if $x(0) = 1$; $x(1)$ is free.

   (b) Suppose that the system (5)

$$\dot{x}_1(t) = x_2(t) - x_1(t)$$

$$\dot{x}_2(t) = -2x_1(t) - 3x_2(t) + u(t)$$

is to be controlled to minimise the performance measure

$$J(x, u) = \frac{1}{2}\int_{t_0}^{t_f} [x_1^2(t) + x_2^2(t) + u^2(t)]dt$$

Find the necessary conditions for optimal control.

PART B

4. In which context Pontryagin's Minimum Principle is used in an optimal control problem? State and prove Pontryagin's Minimum principle. (9)

5. (a) Compare Bang Bang control and Bang off Bang control (4)

   (b) Derive the conditions for the existence of singular intervals in linear Time Optimal Problems. (5)

6. (a) The system (5)

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -x_2(t) + u(t)$$

   is to be controlled to minimize the performance measure

$$J(u) = \int_{t_0}^{t_f} \frac{1}{2}[x_1{}^2(t) + u^2(t)]dt$$

   where the constraining equations $x(t_0)$ is specified, the final state $x(t_f)$ is free and t_f is given. The admissible control values are constrained by

$$-1 \leq u(t) \leq 1 \quad for \quad t \in [t_0, t_f]$$
$$-2 \leq x_2(t) \leq 2 \quad for \quad t \in [t_0, t_f]$$

   Find the necessary conditions for optimal control.

   (b) What do you mean by minimum energy problem? Explain with the help of an example. How will you find necessary conditions for optimal control for this problem? (4)

PART C

7. (a) A first order discrete time system is described by the difference equation (9)

$$x(k+1) = -0.5x(k) + u(k)$$

   The performance measure to be minimized is

$$J = \sum_{k=0}^{2} |x(k)|$$

   and the admissible states and controls are constrained by

$$-0.2 \leq x(k) \leq 0.2, \qquad k = 0, 1, 2$$

$$-0.1 \leq u(k) \leq 0.1, \qquad k = 0, 1$$

   Carry out the computational steps required to determine the optimal control law by using dynamic programming. Quantize both $u(k)$ and $x(k)$ in steps of 0.1 about zero and use linear interpolation. What is the optimal control sequence for an initial taste value of 0.2.

(b) State and explain Principle of Optimality. (3)

8. (a) What is the use of Hamilton Jacobi Bellman(HJB) in optimal control theory? (3)

(b) Given a first order system (9)

$$\dot{x}(t) = -2x(t) + u(t)$$

with performance measure

$$J = \frac{1}{2}x^2(t_f) + \frac{1}{2}\int_0^{t_f}[x^2(t) + u^2(t)]dt$$

Find the optimal control using HJB equation.

9. Write notes on

(a) Recurrence relation in dynamic programming (6)

(b) Continuous linear regulator problem (6)