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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

Branch: Mechanical

Stream: Machine Design

1.

Course Code & Name: 01MA6011 Special Functions, Partial Differential Equations and Tensors.

Answer *any two full* questions from *each* part

Limit answers to the required points.

Max. Marks: 60 Duration: 3 hours

PART A

1. a. If $\boldsymbol{r} = xi + y\boldsymbol{j} + z\boldsymbol{k}$, show that $grad\, r = \frac{r}{r}$ and $grad \left(\frac{1}{r}\right) = -\frac{r}{r^3}$ (5 marks)

   b. Evaluate $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region by $y = \sqrt{x}$ and $y = x^2$ (4 marks)

2. a. Find $div\, \boldsymbol{F}$ and $curl\, \boldsymbol{F}$ where $\boldsymbol{F} = grad\,(x^3 + y^3 + z^3 - 3xyz)$ (5 marks)

   b. A covariant tensor has components $2x - z, x^2y \; and \; yz$ in rectangular coordinates. Find its covariant components in spherical coordinates. (4 marks)

3. a. Find the components of the first and second fundamental tensors in cylindrical coordinates. (4 marks)

   b. If $(ds)^2 = (dr)^2 + r^2(d\theta)^2 + r^2 sin^2\theta(d\varphi)^2$, find the values of $[22,1] \; and \; \{{}^{\,1}_{22}\}$. (5 marks)

PART B

4. a. Show that the integral equation

$$y(x) = \frac{1}{2}x^2 + \int_0^x y(t).\, t(t - x)dt$$

is equivalent to the differential equation $y''(x) + xy(x) = 1; y(0) = y'(0) = 0$

(4 marks)

b. Solve

$y'(x) = 2 + \int_0^x y(t).\cos 2(x-t)\,dt, y(0) = 1$ (5 marks)

5. a. By the method of successive approximation solve the integral equation

$y(x) = 1 + \lambda \int_0^x xt.y(t)dt$ (4 marks)

b. Reduce to the canonical form the PDE

$u_{xx} + x^2 u_{yy} = 0$ (5 marks)

6. Solve the IBVP using Laplace transform technique

$$u_{xx} = \frac{1}{c^2} u_{tt} - \cos\omega t, 0 \le x < \infty, 0 \le t < \infty;$$

$$u(0,t) = 0, \quad u \text{ is bounded as } t \to \infty,$$

$$u_t(x,0) = u(x,0) = 0$$ (9 marks)

PART C

7. (a) Prove that $J_{\frac{-1}{2}}(x) = \sqrt{\frac{2}{\pi x}}$ Cosx (3 marks)

(b) State and prove the orthogonality property of the Bessel function (5 marks)

© Prove that $\Gamma(\frac{1}{2})=\sqrt{\pi}$ (4 marks)

8. (a) Prove that $J_0+2J_2+2J_4+\ldots\ldots\ldots\ldots = 1$ (4 marks)

(b) State and prove the Rodrigue formula

(5 marks)

© Evaluate $\int_0^{\frac{\pi}{2}} \sin^9\theta \cos^5\theta\, d\theta$ (3 marks)

9. Solve PDE

$$u_{xx} + u_{yy} + 10(x^2 + y^2 + 10) = 0$$

over the square with sides $x = 0, y = 0, x = 3$ *and* $y = 3$ *with* $u = 0$ on the boundary and mesh length 1. (12 marks)