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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M TECH DEGREE EXAMINATION, DECEMBER 2018

Branch: **Mechanical Engineering**

Stream: **Machine Design**

Course Code & Name: 01 MA6011 Special Functions, Partial Differential Equations and Tensors

Answer any two full questions from each part

Limit answers to the required points

Max. Marks: 60

Part A

1(a). Evaluate $\int_S \boldsymbol{F.n}\, dS$ where $F = 4x\boldsymbol{i} - 2y^2\boldsymbol{j} + z^2\boldsymbol{k}$ and S is the surface bounding the region $x^2 + y^2 = 4,\ z = 0\ and\ z = 3.$ (3 marks)

(b).State Divergence theorem (3 marks)

(c). Define: Gradient, Divergence and Curl (3 marks)

2 (a). Using Green's theorem evaluate $\int_C \ y^2dx + x^2dy$ where C is the square with vertices (0,0), (1,0), (1,1) and (0,1) oriented counter clockwise. (6 marks)

(b) Define (i) contravariant tensor of order 1

(ii) covariant tensor of order 1

(iii) mixed tensor of order 2 (3 marks)

3 (a). Find the components of the first and second fundamental tensors in spherical coordinates (6 marks)

(b). Show that the velocity of a fluid at any point is a contravariant tensor of rank 1 (3 marks)

Part B

4 (a). Transform the IVP $y'' + xy' + y = 0,\ y(0) = 1, and\ y'(0) = 0$ into an integral equation. (5 marks)

(b). Solve $3x^2 + \int_0^x y(t)\sin(x-t)\,dt = y(x)$ (4 marks)

5 (a). Solve the integral equation $y(x) = 1 + \lambda\int_0^1 xty(t)dt$ by the method of successive approximations (4 marks)

(b). Reduce to the canonical form $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = e^x$ (5 marks)

6 (a). Using Laplace transforms solve the IBVP

$u_{xx} = \frac{1}{c^2}u_{tt} - \cos\omega t,\ 0 \le \mathrm{x} < \infty,\ 0 \le \mathrm{t} < \infty,\ \mathrm{u}(0,\mathrm{t}) = 0,$

u is bounded as x tends to ∞, $\mathrm{u_t}(\mathrm{x},0) = \mathrm{u}(\mathrm{x},0) = 0$ (6 marks)

(b). When the second order partial differential equation

$R.u_{xx} + S.u_{xy} + T.u_{yy} + f(x,y,u,u_x,\ u_y) = 0$

is said to be (i) Elliptic (ii) Parabolic and (iii) hyperbolic (3 marks)

Part C

7 (a).Prove that $B(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$ (6 marks)

(b). Deduce that $\Gamma(½) = \sqrt{\pi}$ from $B(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$ (2 marks)

(c). Prove that $J_{½}(x) = \sqrt{\frac{2}{\pi x}}.sinx$ (4 marks)

8 (a).Prove that $\cos(xsin\theta) = J_0(x) + J_2(x).2cos2\theta + J_4(x).2cos4\theta + \cdots$ (4 marks)

(b). Express the polynomial $x^3 - 5x^2 + x + 2$ in terms of Legendre's polynomials

(3 marks)

(c).State and prove the orthogonality property of Legendre polynomials (5 marks)

9. Using Crank Nicholson method solve $u_{xx} = 16u_t, 0 < \mathrm{x} < 1, \mathrm{t} > 0$

given $\mathrm{u}(\mathrm{x},0) = 0,\ \mathrm{u}(\mathrm{x},\mathrm{t}) = 0, \mathrm{u}(1,\mathrm{t}) = 50\mathrm{t}$. Compute u for 2 steps in t direction taking $h = \frac{1}{4}$. (12 marks)