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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

Branch: Mechanical

Stream: Machine Design

Course Code & Name: 01MA6011 Special Functions, Partial Differential Equations and Tensors.

Answer *any two full* questions from *each* part

Limit answers to the required points.

Max. Marks: 60 Duration: 3 hours

PART A

1. a. Evaluate $\iiint_V \nabla.\boldsymbol{F}dV$ where $F = (2x^2 - 3z)\boldsymbol{i} - 2xy\boldsymbol{j} - 4x\boldsymbol{k}$ and V is bounded by the planes $x = 0, y = 0, z = 0$ *and* $2x + 2y + z = 4$ (5 marks)

   b. Show that $F = (x^2 + xy^2)\boldsymbol{i} + (y^2 + yx^2)\boldsymbol{j}$ is conservative and find its scalar potential. (4 marks)

2. a. Verify Stoke's theorem for $\boldsymbol{F} = (x^2 + y^2)\boldsymbol{i} - 2xy\boldsymbol{j}$ taken round the rectangle bounded by the lines $x = a, x = -a, y = 0$ *and* $y = b$ (6 marks)

   b. A covariant tensor has components $xy, 2y - z^2$ *and* xz in rectangular coordinates. Find its covariant components in spherical coordinates. (3 marks)

3. a. Prove that (i) a symmetric tensor of order 2 has only $\frac{1}{2}n(n+1)$ different components and (ii) a skew symmetric tensor of order 2 has only $\frac{1}{2}n(n-1)$ different non-zero components (5 marks)

   b. Find the components of the metric tensor and conjugate tensor in spherical coordinates (4 marks)

PART B

4. a. Convert the differential equation $y''(x) - 3y'(x) + 2y(x) = 5\sin x$; $y(0) = 1, y'(0) = -2$ into an integral equation (4 marks)

   Solve the integral equation

   b. $y(x) = 3x^2 + \int_0^x y(t).\sin(x-t)\,dt$ (5 marks)

5. a. By the method of successive approximation solve the integral equation

$y(x) = 1 + x + \int_0^x y(t).(x - t)dt$ (3 marks)

b. Reduce to the canonical form the PDE

$y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$ (6 marks)

6. a. Solve the IBVP using Laplace transform technique

$$u_t = u_{xx}, 0 < x < 1, t > 0;$$

$$u(0,t) = 1, u(1,t) = 1, t > 0;$$

$u(x,0) = 1 + sin\pi x, 0 < x < 1$ (6 marks)

b. Prove that L(u_{tt}) $= s^2U(x,s) - su(x,0) - u_t(x,0)$ (3 marks)

PART C

7. (a) Show that $(1 - 2xt + t^2)^{\frac{-1}{2}} = \sum_{n=0}^{\infty} t^n P_n(x)$ (7 marks)

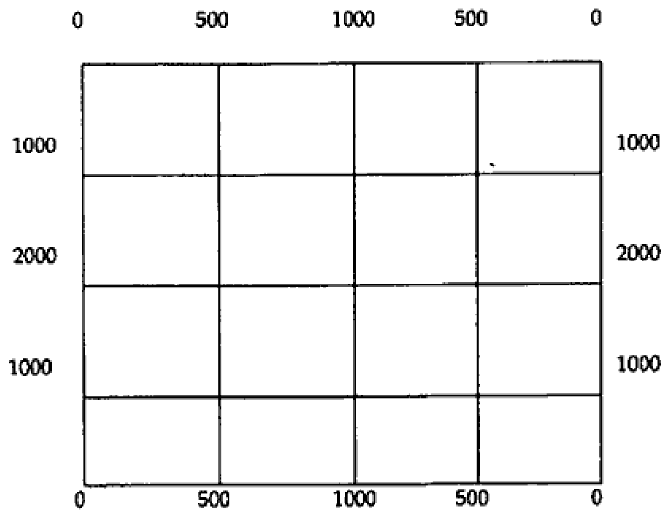
(b) Prove that $\Gamma(\frac{1}{2})=\sqrt{\pi}$ (5 marks)

8. (a) Prove that $sin(xCos\theta) = 2[J_1 Cos\theta - J_3 cos3\theta + J_5 cos5\theta - \cdots ....]$ (4 marks)

(b) Prove that $J_{-n}(x) = (-1)^n J_n(x)$ (4 marks)

© Express the polynomial$x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. (4 marks)

9. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh and boundary values as shown below



(12 marks)