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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2015

Mechanical Engineering

(Machine Design)

01ME6105 Continuum Mechanics

Max. Marks : 60 Duration: 3 Hours

Answer any two full questions from each module.

Part A (Modules I & II) – Max marks:18

1. (a) Given a continuum, where the stress state is known at one point and is represented by the Cauchy stress tensor components $\sigma_{ij} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ Pa. find the principal stresses and principal directions *(6 marks)*

   (b) Show that (i) $\delta_{3p}v_p = v_3$ (ii) $\delta_{3i}A_{ji} = A_{j3}$ (iii) $\delta_{i2}\delta_{j3}A_{ij} = A_{23}$ *(3 marks)*

2. (a) Prove the identity $\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$ *(5 marks)*

   (b) The Cauchy stress tensor at point P is given by $\sigma_{ij} = \begin{bmatrix} 5 & 6 & 7 \\ 6 & 8 & 9 \\ 7 & 9 & 2 \end{bmatrix}$ GPa. Obtain the deviatoric and volumetric parts of the tensor. *(4 marks)*

3. (a) Prove the vector identity $u \times (v \times w) = (u.w)v - (u.v)w$ *(5 marks)*

   (b) The stress state at one point is represented by the Cauchy stress components $\sigma_{ij} = \begin{bmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{bmatrix}$. where a, b, c are constants and σ is the value of the stress. Determine the constants such that the traction vector on the octahedral plane is zero. *(4 marks)*

Part B (Modules III & IV) – Max marks:18

4. (a) Given the motion of a body to be $x_i = X_i + 0.2tX_2\delta_{1i}$, for a temperature field given by $\theta = 2x_1 + (x_2)^2$, find the material description of temperature and rate of change of temperature of a particle, which at time t = 0 was at the place (0,1,0). *(5 marks)*

   (b) Deduce the equilibrium equations from linear momentum principle. *(4 marks)*

5. (a) Obtain the infinitesimal strain tensor and the infinitesimal spin tensor for the following displacement field ; $u_1 = x_1^2$, $u_2 = x_1x_2$, $u_3 = 0$ *(5 marks)*

   (b) Prove the symmetry of stresses $\sigma_{ij} = \sigma_{ji}$ using principle of conservation of angular momentum *(4 marks)*

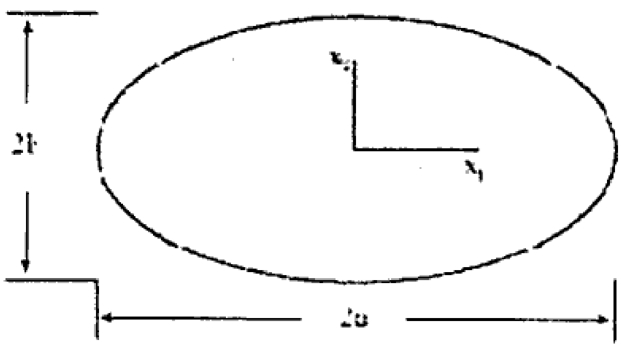
Turn over

6. (a) Obtain the Lagrangian and Eulerian forms of continuity equation. *(4 marks)*
   (b) The deformation of a body is given by $u_1 = (3X_1^2 + X_2)$, $u_2 = (2X_2^2 - X_3)$, and $u_3 = (4X_3^2 + X_1)$. Compute the vector into which the vector $10^{-3}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ passing through the point (1,1,1) in the reference configuration is deformed. *(5 marks)*

Part C (Modules V & VI) – Max marks:24

7. (a) From linear elastic constitutive relation for isotropic materials, deduce the strain-stress relation $\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\,\delta_{ij}$. *(6 marks)*
   (b) Given an isotropic linear elastic material, whose elastic properties are E = 71 GPa, G = 26.6 GPa, find the strain tensor components and the strain energy density at the point in which the stress state, in Cartesian basis is represented by
   $$\sigma_{ij} = \begin{bmatrix} 20 & -4 & 5 \\ -4 & 0 & 10 \\ 5 & 10 & 15 \end{bmatrix} GPa$$ *(6 marks)*

8. Determine the stresses and the angle of twist for a solid elliptical shaft of the dimensions shown when subjected to end couples M_t. *(12 marks)*



9. Consider a special stress function having the form $\Phi = B_2x_1x_2 + D_4x_1x_3$. Show that this stress function may be adapted to solve for the stresses in an end-loaded cantilever beam shown in the sketch. Assume the body forces are zero for this problem. *(12 marks)*