2 Pages **D**

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

Mechanical Engineering

Machine Design

01ME6105 CONTINUUM MECHANICS

Answer *any two full* questions from *each* part

Limit answers to the required points.

Max. Marks: 60 Duration: 3 hours

PART A

1. a. Expand $\nabla \cdot \sigma$ where ∇ is the vector differential operator and σ is a second order tensor. (3 marks)

   b. Write the dot product of two vectors in terms of Kronecker delta symbol. Prove that $\nabla \cdot \vec{a} \times \vec{b} = \vec{b} \cdot (\nabla \times \vec{a}) - \vec{a} \cdot (\nabla \times \vec{b})$ (3 marks)

   c. What are Levi-Civita symbols? Write down the cross product of two vectors in terms of Levi-Civita symbols. Write down the expanded form of cross product of two vectors. (3 marks)

2. a. Write the expanded form of the following equation given in indexed form $\frac{\partial u_i}{\partial t} + u_j \frac{\partial (u_i u_j)}{\partial x_j} = \frac{-\partial P}{\partial x_i} + \frac{\partial^2 u_i}{\partial x_j x_j}$ (3 marks)

   b. Write down the $\varepsilon - \delta$ identity (1 marks)

   c. A vector field is given by $\hat{u} = (x^2+3)\hat{i} + 3y^2 z\hat{j} + (x+3z)\hat{k}$ Let $I = \int_s \hat{u} \cdot \hat{n}\, ds$ is a scalar connected with a volume "v" having surface "s". Determine the value of I per unit volume when "v" is an infinitesimal small volume around the point (1,2,3) (5 marks)

3. a. The state of stress at a point is given by the Cartisian stress tensor $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ kPa. Find (a) the three stress invariants, (b) Characteristic equation, (c) Principal stresses. (4 marks)

b. Find the unit normals to principal planes corresponding to the state of stress given in question (3a). Write a matrix of coordinate transformation consisting of the vectors of unit normals to principal planes. Use this matrix to transform the stress matrix given in question (3a). (5 marks)

PART B

4 a. Define the terms (i) Stretch (ii) Engineering strain (iii) True strain (iv) Lagrangian strain (v) Eulerian strain and (vi) Logarithmic strain (3 marks)

b. Derive the material derivative operator (6 marks)

5 a. Obtain the Lagrangian form of equation of mass balance. (4 marks)

b. A function $f(x)$ is defined on a line with $x \in [0,1]$. Let $\int_p \left(\frac{df(x)}{dx} - f(x)\right) = 0$ where "p" is any small portion of the above line and $f(0)=1$. Find the values of $f(0.25)$, $f(0.5)$, $f(0.75)$ and $f(1)$ (5 marks)

6. a. Prove that stress tensor is symmetric (5 marks)

b. What is deformation gradient tensor? Obtain an expression for Lagrangian strain in terms of deformation gradient tensor (4 marks)

PART C

7. a. What is monoclinic material? Write down the transformation matrix by selecting the Cartesian coordinate directions for the directions of symmetry. Write down the number of independent elastic constants . Write down the structure of the elasticity matrix relating the stress vector and strain vector. (1+1+1+2=5 marks)

b. Discuss the principle of material indifference. (4 marks)

c. Write down the equations Lame's coefficients in terms of Young's modulus and Poisson's ration. (3 marks)

8. a. Derive the Beltrami-Michell equations corresponding to isotropic elasticity. (6 marks)

b. Write down Navier-Stokes equation. Simplify it for the flow of an inviscid fluid. Hence derive the Bernoulli"s equation. (6 marks)

9. a. Write down the stress strain relations of plain strain in polar coordinates. (2 marks)

b. Obtain the solution for stress in a uni axially loaded large plate with a hole. (10 marks)