Reg. No: ................................ **SLOT C**

Name: ..................................

# APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

## FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

# Mechanical Engineering

## Machine Design

## 01ME6103, FINITE ELEMENT METHOD

Time: 3 Hours | Max. Marks: 60

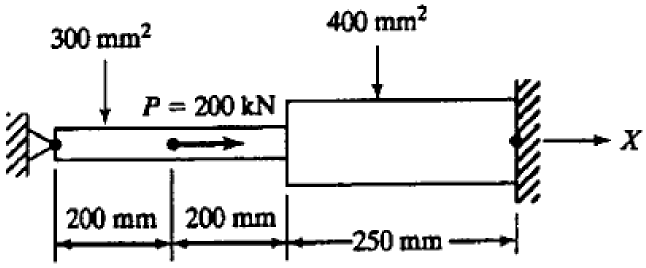
Instructions: Answer any two questions from each part

## PART A

1.
   a. How is patch test done in FEM? 2
   b. What are the properties of Shape function? 2
   c. Derive the stiffness matrix of a beam element 5
2.
   a. Prove that the relation between global stiffness matrix and local stiffness matrix is given by $[K]_{global} = [T]^T [K]_{local} [T]$ where $[T]$ is the transformation matrix. 3
   b. Determine the shape functions for the bar element with end nodes and mid length node shown in Figure. Let the variation of axial displacement inside the element is $u(x) = a_1 + a_2 x + a_3 x^2$.


3

   c. Discuss the different types of refinement in FEM. 3
3. Consider the bar shown in figure. Determine the nodal displacements, element stresses, and support reactions. 9



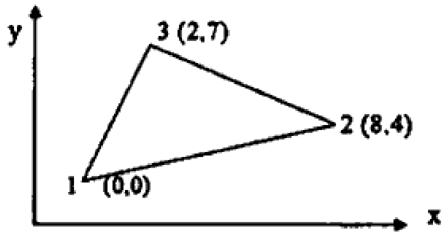

$E = 123 \times 10^3$ N/mm²
(1 kN = 1000 N)

## PART B

4.
   a. Derive the stiffness matric for a CST element using the principle of minimization of potential energy. 7

b. For the element shown in the figure find the displacements at the point (3,5) if the nodal displacement is given by

$[0.0001 \quad -0.004 \quad 0.003 \quad 0.002 \quad -0.002 \quad 0.005]^T$ 2




5.

a. Discuss Neumann, Dirichlet and Robin boundary conditions 3

b. Consider a uniform rod subjected to linearly varying load q=ax. The governing differential equation is given by $AE\frac{d^2u}{dx^2}+ax=0$ with boundary conditions $u(0)=0, AE\frac{du}{dx}\Big|_{x=L}=0$. Solve this equation using weighted residual technique. 6

6. Explain the Galerkin Finite Element Method for a one dimensional problem considering the differential equation $\frac{d^2y}{dx^2}+f(x)=0,\ a\le x\le b$, subject to the boundary conditions $y(a)=y_a,\ y(b)=y_b$ 9

## PART C

7. Discuss the use of axisymmetric elements in FEM and derive the stiffness matrix of any axisymmetric element. 12

8.

a. Draw an isoparametric 8 node rectangular element and write the shape functions 2

b. Evaluate the integrals $I=\int_{-1}^{1}(3^x-x)dx$ using three point Gaussian quadrature where the integration points and weights are -0.77459, 0, 0.77459 and $\frac{5}{9},\frac{8}{9},\frac{5}{9}$ respectively. 3

c. Show that the Jacobian for a four node isoparametric quadrilateral element is given by $[J]=\frac{1}{8}[x_1 \quad x_2 \quad x_3 \quad x_4]\begin{bmatrix}0 & 1-t & t-s & s-1\\ -1+t & 0 & s+1 & -s-t\\ s-t & -1-s & 0 & 1+t\\ 1-s & s+t & -1-t & 0\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\\ y_4\end{bmatrix}$ 7

9.

a. Derive the consistent mass matrix for a beam element. 5

b. Discuss any Central difference technique for transient analysis 4

c. Discuss the Newton Raphson technique for nonlinear problems 3